Closed string noncommutativity in the weakly curved background*

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Abstract

We consider the closed bosonic string moving in the weakly curved background. Using T-duality transformation laws we calculate the Poisson brackets of the coordinates in the T-dual space assuming that initial theory is geometric one, which means that standard Poisson algebra is obeyed. The result is that the commutative initial theory is equivalent to the non-commutative T-dual theory. All noncommutativity parameters are infinitesimal and proportional to the $B_{\mu\nu\rho}$, field strength of Kalb-Ramond field $B_{\mu\nu}$. In addition we find the algebra of the T-dual winding numbers and momenta in terms of the winding numbers and momenta of the initial theory.

1. Introduction

In order to obtain noncommutativity in the open string case it is enough to consider the open string in the presence of the *constant* gravitational $G_{\mu\nu}$ and Kalb-Ramond field $B_{\mu\nu}$ and use the boundary conditions [1, 2]. Treating boundary conditions as canonical constraints and solving them, one gets the initial coordinates expressed in terms of the Ω even effective coordinates and momenta, where Ω is world-sheet parity transformation $\Omega : \sigma \to -\sigma$. Because effective variables have nonzero Poisson bracket (PB), the PB between initial coordinates is also nonzero. The noncommutativity parameter is proportional to the Kalb-Ramond field $B_{\mu\nu}$.

There is one interesting thing which we noted in the open string case. The effective metric and the noncommutativity parameter are (up to some constants) the backgroud fields of the T-dual theory. As we know T-dual theory is physically equivalent to the initial one in the sense they have the same degrees of freedom - one at the scale R and the T-dual one at the scale $1/R$. The mathematical realization of the T-duality goes through Buscher procedure [3]. As a result of the procedure we get the relation between initial and T-dual variables which we call *transformation laws.*

* Work supported in part by the Serbian Ministry of Education and Science, under contract number No.171031.

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The closed strings do not have endpoints, so in the constant background there are no boundary conditions. To obtain noncommutativity in the closed string case we have to use T-duality as a helping tool. But, in the constant background case, T-duality relates σ-derivatives of the coordinates of one theory with the momenta of its T-dual one. Assuming that momenta of the initial theory commute (geometric theory) it follows that the T-dual coordinates commute as well. Consequently, in the constant background case there is no closed string non-commutativity.

It is obvious that T-duality is just one part of the solution in order to get the closed string noncommutativity. The second part is coordinate dependent background obeying the space-time field equations [4, 5]. Considering the closed string in the constant gravitational field $G_{\mu\nu}$ and Kalb-Ramond field depending on one coordinate, the closed string non-commutativity was first observed in the paper [6], and investigated further in [7, 8, 9]. In these articles 3-torus is considered, where $B_{\mu\nu}$ depends on one coordinate and T-dualization is performed along two other coordinates (isometry directions) using standard Buscher procedure [3].

One can ask if it is possible to do that in the background where $B_{\mu\nu}$ depends on all space-time coordinates. The answer is affirmative but in order to achieve that we have to use the generalized T-duality procedure presented in details in [10] and to apply it to the weakly curved background. The weakly curved background used in the present article is defined by constant gravitational $G_{\mu\nu} = const$ and the linear Kalb-Ramond field $B_{\mu\nu} = b_{\mu\nu} + \frac{1}{3}B_{\mu\nu\rho}x^\rho$, where the field strength $B_{\mu\nu\rho}$ is supposed to be infinitesimal. Such background obeys space-time field equations [4, 5] in the linear approximation in $B_{\mu\nu\rho}$.

We perform the generalized T-dualization procedure [10] along all the coordinates and obtain the T-duality transformation law, $\partial_\pm y_\mu = \partial_\pm y_\mu(\partial_\pm x)$, where $\partial_\pm$ are world-sheet partial derivatives. Using canonical formalism, the T-dual coordinates are expressed in terms of the original variables, $y'_\mu \cong \frac{1}{\kappa}\pi_\mu - \beta^0_\mu[x]$, where π_μ are canonically conjugated momenta to the coordinates x^μ. The infinitesimal expression β^0_μ is the correction in comparison to the flat background case. Assuming that the coordinates and momenta of the original theory satisfy standard Poisson algebra (initial theory is geometric one), we get the coordinate noncommutativity relations in the T-dual picture. In addition, we obtain the complete algebra of the T-dual winding numbers and momenta.

2. Generalized T-duality and noncommutativity

We consider the closed bosonic string moving in the D-dimensional space-time described by the action

$$S[x] = \kappa \int_\Sigma d^2\xi \; \partial_+ x^\mu \left(B_{\mu\nu}[x] + \frac{1}{2} G_{\mu\nu}[x] \right) \partial_- x^\nu \,, \tag{1}$$

where the light-cone coordinates are defined as $\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$ and the corresponding derivatives $\partial_\pm = \partial_\tau \pm \partial_\sigma$. In order to keep conformal invariance on the quantum level, the background fields have to obey the following one-loop consistency conditions [4, 5]

$$R_{\mu\nu}-\frac{1}{4}B_{\mu\rho\sigma}B_{\nu}{}^{\rho\sigma}=0,\quad D_\rho B^\rho{}_{\mu\nu}=0\,. \tag{2}$$

Here $B_{\mu\nu\rho}=\partial_\mu B_{\nu\rho}+\partial_\nu B_{\rho\mu}+\partial_\rho B_{\mu\nu}$ is the field strength of the field $B_{\mu\nu}$, and $R_{\mu\nu}$ and D_μ are Ricci tensor and the covariant derivative with respect to the space-time metric.

The solution of the equations in the first order in $B_{\mu\nu\rho}$, so called the weakly curved background, [7, 10, 11, 12], is defined by

$$\begin{aligned} G_{\mu\nu}[x] &= const,\\ B_{\mu\nu}[x] &= b_{\mu\nu}+h_{\mu\nu}[x]=b_{\mu\nu}+\frac{1}{3}B_{\mu\nu\rho}x^\rho, \qquad b_{\mu\nu},B_{\mu\nu\rho}=const. \end{aligned} \tag{3}$$

Here, the field strength $B_{\mu\nu\rho}$ is infinitesimal.

Applying the generalized T-dualization procedure [10] on the closed string propagating in the weakly curved background, we obtain the T-dual action

$${}^\star S[y]=\frac{\kappa^2}{2}\int d^2\xi\ \partial_+y_\mu\Theta_-^{\mu\nu}[\Delta V[y]]\partial_-y_\nu, \tag{4}$$

where

$$\begin{aligned} \Theta_\pm^{\mu\nu} &\equiv -\frac{2}{\kappa}(G_E^{-1}\Pi_\pm G^{-1})^{\mu\nu}=\theta^{\mu\nu}\mp\frac{1}{\kappa}(G_E^{-1})^{\mu\nu}\,,\\ G_{E\mu\nu} &\equiv G_{\mu\nu}-4(BG^{-1}B)_{\mu\nu}\,,\quad \Pi_{\pm\mu\nu}=B_{\mu\nu}\pm\frac{1}{2}G_{\mu\nu}\,. \end{aligned} \tag{5}$$

The argument ΔV is defined nonlocally as

$$\Delta V^\mu[y]=-\kappa\theta_0^{\mu\nu}\Delta y_\nu+(g^{-1})^{\mu\nu}\Delta\tilde{y}_\nu\,, \tag{6}$$

where

$$\Delta y_\mu=\int_P(d\tau\dot{y}_\mu+d\sigma y'_\mu)=y_\mu(\xi)-y_\mu(\xi_0),\quad \Delta\tilde{y}_\mu=\int_P(d\tau y'_\mu+d\sigma\dot{y}_\mu), \tag{7}$$

and

$$g_{\mu\nu}=G_{\mu\nu}-4(bG^{-1}b)_{\mu\nu},\quad \theta_0^{\mu\nu}=-\frac{2}{\kappa}(g^{-1}bG^{-1})^{\mu\nu}\,. \tag{8}$$

It is obvious from the definitions (7) that these two coordinates are related by the following expressions, $\dot{y}_\mu=\tilde{y}'_\mu$, $y'_\mu=\dot{\tilde{y}}_\mu$.

The transformation laws connecting initial and T-dual coordinates play the key role in our considerations. To be more precise, we obtain from T-dualization procedure the relations between world-sheet derivatives of the initial and T-dual coordinates

$$\partial_\pm x^\mu\cong-\kappa\Theta_\pm^{\mu\nu}[\Delta V]\Big[\partial_\pm y_\nu\pm2\beta_\nu^\mp[V]\Big], \tag{9}$$

where

$$\beta^{\pm}_{\mu}[x] = \frac{1}{2}(\beta^0_\mu \pm \beta^1_\mu) = \mp\frac{1}{2}h_{\mu\nu}[x]\partial_\mp x^\nu\,,$$
$$\beta^0_\mu[x] = h_{\mu\nu}[x]x'^\nu,\; \beta^1_\mu[x] = -h_{\mu\nu}[x]\dot{x}^\nu\,. \tag{10}$$

Because we use the canonical formalism, we must have these transformation laws in the canonical form

$$x'^\mu \cong \frac{1}{\kappa}\,{}^\star\pi^\mu - \kappa\theta_0^{\mu\nu}\beta^0_\nu[V], \tag{11}$$
$$\pi_\mu \cong \kappa y'_\mu + \kappa\beta^0_\mu[V]\,, \tag{12}$$

where π_μ and ${}^\star\pi^\mu$ are canonically conjugated momenta to the coordinates x^μ and y_μ, respectively. It is shown in Ref. [10] that the T-dual of the T-dual action is the initial one. If we want to have T-dual coordinates in terms of the initial ones, we just have to invert the relation (9)

$$\partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu}[\Delta x]\partial_\pm x^\nu \mp 2\beta^\mp_\mu[x]. \tag{13}$$

The canonical form of the T-dual transformations is

$$y'_\mu \cong \frac{1}{\kappa}\pi_\mu - \beta^0_\mu[x], \tag{14}$$
$${}^\star\pi^\mu \cong \kappa x'^\mu + \kappa^2\theta_0^{\mu\nu}\beta^0_\nu[x]. \tag{15}$$

Our intention is to calculate the PB's of the T-dual variables y_μ and $\tilde{y}_\mu$ using PB algebra of the initial variables. Consequently, we assume that initial theory is geometric which means that coordinates x^μ and momenta π_ν satisfy standard PB algebra

$$\{x^\mu(\sigma),\pi_\nu(\bar\sigma)\} = \delta^\mu_\nu\delta(\sigma-\bar\sigma),\quad \{x^\mu(\sigma),x^\nu(\bar\sigma)\}=0,\quad \{\pi_\mu(\sigma),\pi_\nu(\bar\sigma)\}=0. \tag{16}$$

In this article we will calculate, besides already mentioned PB algebra of the T-dual coordinates, also the algebra of the T-dual winding numbers and momenta. For both purposes, the first step is introducing the quantity

$$\Delta Y_\mu(\sigma,\sigma_0) = \int_{\sigma_0}^{\sigma} d\eta\, Y'_\mu(\eta) = Y_\mu(\sigma) - Y_\mu(\sigma_0), \tag{17}$$

where $Y_\mu = (y_\mu, \tilde{y}_\mu)$. The second step is to calculate their PB's. It is obvious that key relation which we have to calculate is PB between σ derivatives of Y's. When we calculate it in three possible cases it turns out that it can be written in the form

$$\{X'_\mu(\sigma), Y'_\nu(\bar\sigma)\} \cong K'_{\mu\nu}(\sigma)\delta(\sigma-\bar\sigma) + L_{\mu\nu}(\sigma)\delta'(\sigma-\bar\sigma)\,. \tag{18}$$

Integrating this relation by parts over σ and $\bar{\sigma}$, after straightforward calculation, we extract PB we are searching for

$$\{X_\mu(\tau,\sigma), Y_\nu(\tau,\bar{\sigma})\} \cong -\left[K_{\mu\nu}(\sigma) - K_{\mu\nu}(\bar{\sigma}) + L_{\mu\nu}(\bar{\sigma})\right]\theta(\sigma-\bar{\sigma})\,, \tag{19}$$

where $\theta(\sigma)$ is the step function defined as

$$\theta(\sigma) = \begin{cases} 0 & \text{if } \sigma = 0 \\ 1/2 & \text{if } 0 < \sigma < 2\pi, \\ 1 & \text{if } \sigma = 2\pi \end{cases} \quad \sigma \in [0, 2\pi]. \tag{20}$$

This is a general form of the relation. Using transforation laws we calculate PB's in three cases: $\{y'_\mu(\sigma), y'_\nu(\bar{\sigma})\}$, $\{y'_\mu(\sigma), \tilde{y}'_\nu(\bar{\sigma})\}$ and $\{\tilde{y}'_\mu(\sigma), \tilde{y}'_\nu(\bar{\sigma})\}$, and express them in the form of (18). Reading the corresponding values of K and L and using (19), we get the noncommutativity relations for T-dual closed string coordinates

$$\{y_\mu(\sigma), y_\nu(\bar{\sigma})\} \cong -\frac{1}{\kappa}B_{\mu\nu\rho}[x^\rho(\sigma) - x^\rho(\bar{\sigma})]\theta(\sigma-\bar{\sigma}), \tag{21}$$

$$\{y_\mu(\sigma), \tilde{y}_\nu(\bar{\sigma})\} \cong -\Big\{\frac{1}{\kappa}B_{\mu\nu\rho}[\tilde{x}^\rho(\sigma) - \tilde{x}^\rho(\bar{\sigma})] - \frac{3}{2\kappa}\Gamma^E_{\rho,\mu\nu}[x^\rho(\sigma) - x^\rho(\bar{\sigma})]$$
$$+\frac{1}{\kappa}g_{\mu\nu} - \frac{3}{2\kappa}\Gamma^E_{\rho,\mu\nu}\,x^\rho(\bar{\sigma})\Big\}\theta(\sigma-\bar{\sigma}), \tag{22}$$

$$\{\tilde{y}_\mu(\sigma), \tilde{y}_\nu(\bar{\sigma})\} \cong -\Big\{-\frac{1}{\kappa}\Big[B_{\mu\nu\rho} - 6g_{\mu\alpha}Q^{\alpha\beta}{}_\rho g_{\beta\nu}\Big][x^\rho(\sigma) - x^\rho(\bar{\sigma})] \tag{23}$$
$$+\Big[-\frac{3}{2\kappa}\Big(\Gamma^E_{\mu,\nu\rho} - \Gamma^E_{\nu,\mu\rho}\Big) + \frac{4}{\kappa}B_{\mu\nu\sigma}(G^{-1}b)^\sigma{}_\rho\Big][\tilde{x}^\rho(\sigma) - \tilde{x}^\rho(\bar{\sigma})]\Big\}\theta(\sigma-\bar{\sigma})\,,$$

where

$$\tilde{x}'^\mu = \frac{1}{\kappa}(G^{-1})^{\mu\nu}\pi_\nu + 2(G^{-1}B)^\mu{}_\nu x'^\nu\,. \tag{24}$$

Here the infinitesimal fluxes are defined as

$$\Gamma^E_{\mu,\nu\rho} = \frac{1}{2}\Big(\partial_\nu G^E_{\mu\rho} + \partial_\rho G^E_{\mu\nu} - \partial_\mu G^E_{\nu\rho}\Big) = -\frac{4}{3}\Big(B_{\mu\sigma\nu}(G^{-1}b)^\sigma{}_\rho + B_{\mu\sigma\rho}(G^{-1}b)^\sigma{}_\nu\Big), \tag{25}$$

$$Q^{\mu\nu}{}_\rho = -\frac{1}{3}\Big[(g^{-1})^{\mu\sigma}(g^{-1})^{\nu\tau} - \kappa^2\theta_0^{\mu\sigma}\theta_0^{\nu\tau}\Big]B_{\sigma\tau\rho}. \tag{26}$$

For $\sigma = \bar{\sigma}$ we obtain that all PB's vanish, and consequently, coordinates commute. Also we can consider $\sigma = \bar{\sigma} + 2\pi$, which is the same point on the world-sheet as our first choice $\sigma = \bar{\sigma}$. Taking $\sigma = \bar{\sigma} + 2\pi$, three noncommutativity relations take the form

$$\{y_\mu(\sigma+2\pi), y_\nu(\sigma)\} \cong -\frac{2\pi}{\kappa}B_{\mu\nu\rho}N^\rho\,, \tag{27}$$

$$\{y_\mu(\sigma+2\pi), \tilde{y}_\nu(\sigma)\} + \{y_\mu(\sigma), \tilde{y}_\nu(\sigma+2\pi)\} \cong -\frac{4\pi}{\kappa^2} B_{\mu\nu\rho} p^\rho$$
$$+ \quad \frac{\pi}{\kappa}\left(3\Gamma^E_{\rho,\mu\nu} - 8B_{\mu\nu\lambda} b^\lambda{}_\rho\right) N^\rho, \tag{28}$$

and

$$\{\tilde{y}_\mu(\sigma+2\pi), \tilde{y}_\nu(\sigma)\} \cong$$
$$\cong \quad \frac{2\pi}{\kappa}\left[-B_{\mu\nu\rho} - 6g_{\mu\alpha}Q^{\alpha\beta}{}_\rho g_{\beta\nu} + 2B_{\mu\nu}{}^\lambda g_{\lambda\rho} + 3\left(\Gamma^E_{\mu,\nu\lambda} - \Gamma^E_{\nu,\mu\lambda}\right) b^\lambda{}_\rho\right] N^\rho$$
$$+ \quad \frac{\pi}{\kappa^2}\left[3\left(\Gamma^E_{\mu,\nu\rho} - \Gamma^E_{\nu,\mu\rho}\right) p^\rho - 8B_{\mu\nu\lambda} b^\lambda{}_\rho\right] p^\rho\,, \tag{29}$$

where $N^\mu = \frac{1}{2\pi}\left[x^\mu(\sigma+2\pi) - x^\mu(\sigma)\right]$ is winding number of the initial coordinates and

$$p_\mu = \frac{1}{2\pi}\int_\sigma^{\sigma+2\pi} d\eta \pi_\mu(\eta)\,, \tag{30}$$

is mean value of the momentum π_μ. Note that all three PB's are proportional to the Kalb-Ramond field strength which means they are infinitesimal.

In addition we can obtain the algebra of the T-dual winding number and momenta defined as

$$\Delta y_\mu(2\pi, 0) = 2\pi^\star N_\mu\,, \quad \Delta\tilde{y}_\mu(2\pi, 0) = 2\pi^\star P_\mu\,, \tag{31}$$

while we introduced earlier

$$\Delta x^\mu(2\pi, 0) = 2\pi N^\mu\,, \quad \Delta\tilde{x}^\mu(2\pi, 0) = 2\pi P^\mu\,. \tag{32}$$

Using (17), (18), transformation laws and above definitions we have

$$\{{}^\star N_\mu, {}^\star N_\nu\} = \frac{1}{\pi\kappa} B_{\mu\nu\rho} N^\rho\,, \tag{33}$$

$$\{{}^\star N_\mu, {}^\star P_\nu\} = \frac{1}{\pi\kappa} B_{\mu\nu\rho} P^\rho - \frac{3}{4\pi\kappa}\Gamma^E_{\rho,\mu\nu} N^\rho\,, \tag{34}$$

$$\{{}^\star P_\mu, {}^\star P_\nu\} \quad = \quad -\frac{1}{\pi\kappa}\left(B_{\mu\nu\rho} - 6g_{\mu\alpha}Q^{\alpha\beta}{}_\rho g_{\beta\nu}\right) N^\rho$$
$$+ \quad \frac{1}{\pi}\left[-\frac{3}{2\kappa}(\Gamma^E_{\mu,\nu\rho} - \Gamma^E_{\nu,\mu\rho}) + \frac{4}{\kappa} B_{\mu\nu\sigma}(G^{-1}b)^\sigma{}_\rho\right] P^\rho\,.$$

3. Concluding remarks

In the present article we considered the theory describing the closed bosonic string moving in the weakly curved background and derived the non-commutativity relations using canonical approach.

We applied generalized T-duality procedure and obtained the transformation laws connecting the initial and T-dual variables. They, expressed in the canonical form, have the central role in calculation of the PB's of the T-dual coordinates y_μ and $\tilde{y}_\mu$. Infinitesimal Kalb-Ramond field strength, as a part of the function β_μ, gives the main contribution to the noncommutativity parameters. The result is that we showed the physical equivalence of the commutative initial theory and noncommutative T-dual one in linear approximation in the field strength $B_{\mu\nu\rho}$.

The general structure of the non-commutativity relations is

$$\{Y_\mu(\sigma), Y_\nu(\bar{\sigma})\} = \{F_{\mu\nu\rho}\,[x^\rho(\sigma) - x^\rho(\bar{\sigma})] + \tilde{F}_{\mu\nu\rho}\,[\tilde{x}^\rho(\sigma) - \tilde{x}^\rho(\bar{\sigma})]\}\theta(\sigma - \bar{\sigma})\,, \tag{35}$$

where $Y_\mu = (y_\mu, \tilde{y}_\nu)$ and $F_{\mu\nu\rho}$ and $\tilde{F}_{\mu\nu\rho}$ are the constant and infinitesimally small fluxes. At the same points, for $\sigma = \bar{\sigma}$ all PB's are zero. In the important particular case for $\sigma = \bar{\sigma} + 2\pi$ we get

$$\{Y_\mu(\sigma + 2\pi), Y_\nu(\sigma)\} = 2\pi\left[(F_{\mu\nu\rho} + 2\tilde{F}_{\mu\nu\alpha}b^\alpha_\rho)N^\rho + \frac{1}{\kappa}\tilde{F}_{\mu\nu}{}^\rho p_\rho\right]\,, \tag{36}$$

where N^μ and p_μ are winding numbers and momenta of the original theory. In addition we calculated the PB algebra of the T-dual winding numbers and momenta in terms of the initial ones.

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