Holographic approach for quark-gluon plasma in heavy ion collsions

Irina Aref'eva

Steklov Mathematical Institute, RAN, Moscow

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Outlook

- Physical picture of formation of Quark-Gluon Plasma in heavy-ions collisions
- Why holography?
- Results from holography (fit of experimental data via holography:
  top-down
  bottom-up)
  - Holography description of static QGP
  - Holography description of QGP formation in heavy ions collisions
    - Thermalization time
    - Multiplicity

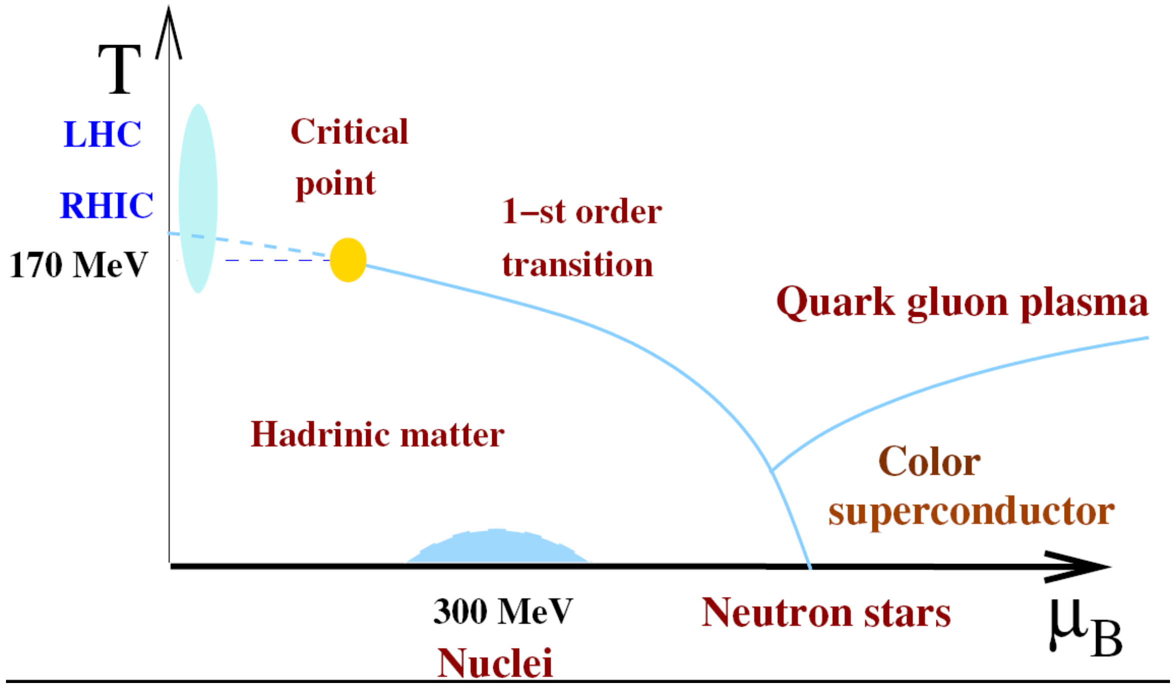
Quark-Gluon Plasma (QGP): a new state of matter

QGP is a state of matter formed from deconfined quarks, antiquarks, and gluons at high temperature

QCD: asymptotic freedom, quark confinement

T increases, or
density increases

nuclear matter → Deconfined phase



Experiments: Heavy Ions collisions produced a medium

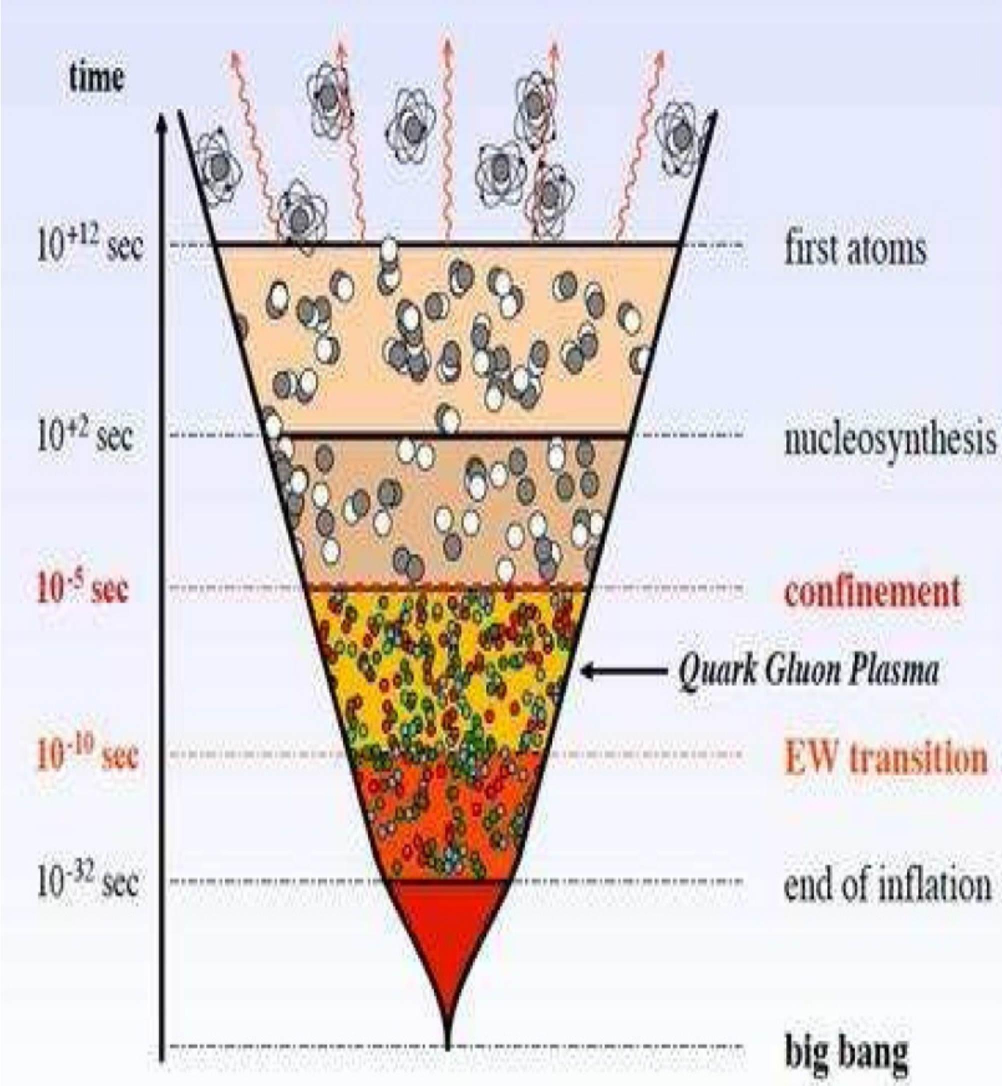
HIC are studied in several **experiments:**

- started in the 1990's at the Brookhaven Alternating Gradient Synchrotron (AGS), $\sqrt{s_{NN}} = 4.75\,GeV$
- the CERN Super Proton Synchrotron (SPS) $\sqrt{s_{NN}} = 17.2\,GeV$
- the Brookhaven Relativistic Heavy-Ion Collider (RHIC) $\sqrt{s_{NN}} = 200\,GeV$
- the LHC collider at CERN. $\sqrt{s_{NN}} = 2.76\,TeV$

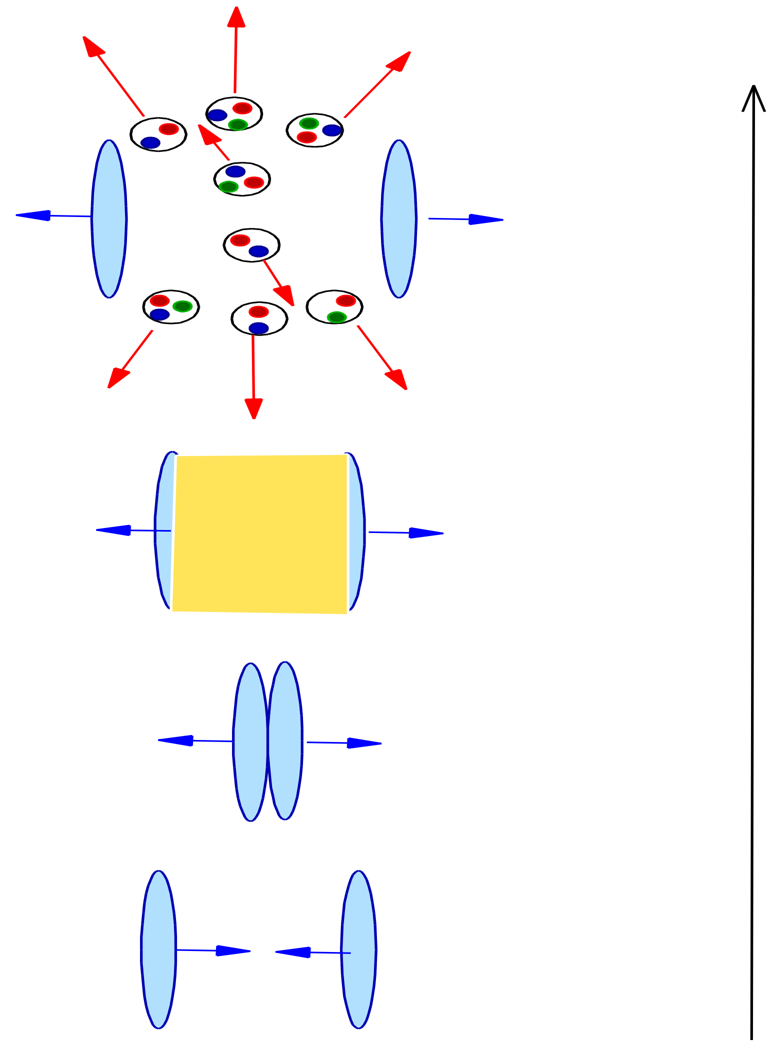
There are strong experimental evidences that RHIC or LHC have created some medium which behaves collectively:

- modification of particle spectra (compared to p+p)
- jet quenching
- high p_T-suppression of hadrons
- elliptic flow
- suppression of quarkonium production

Study of this medium is also related with study of Early Universe



Evolution of the Early Universe



Evolution of a Heavy Ion Collision

$\Delta t \approx 1\ \mathrm{fm/c}$

QGP as a strongly coupled fluid

- **Conclusion from the RHIC and LHC experiments: appearance of QGP (not a weakly coupled gas of quarks and gluons, but a strongly coupled fluid).**

- This makes perturbative methods inapplicable

- The lattice formulation of QCD does not work, since we have to study real-time phenomena.

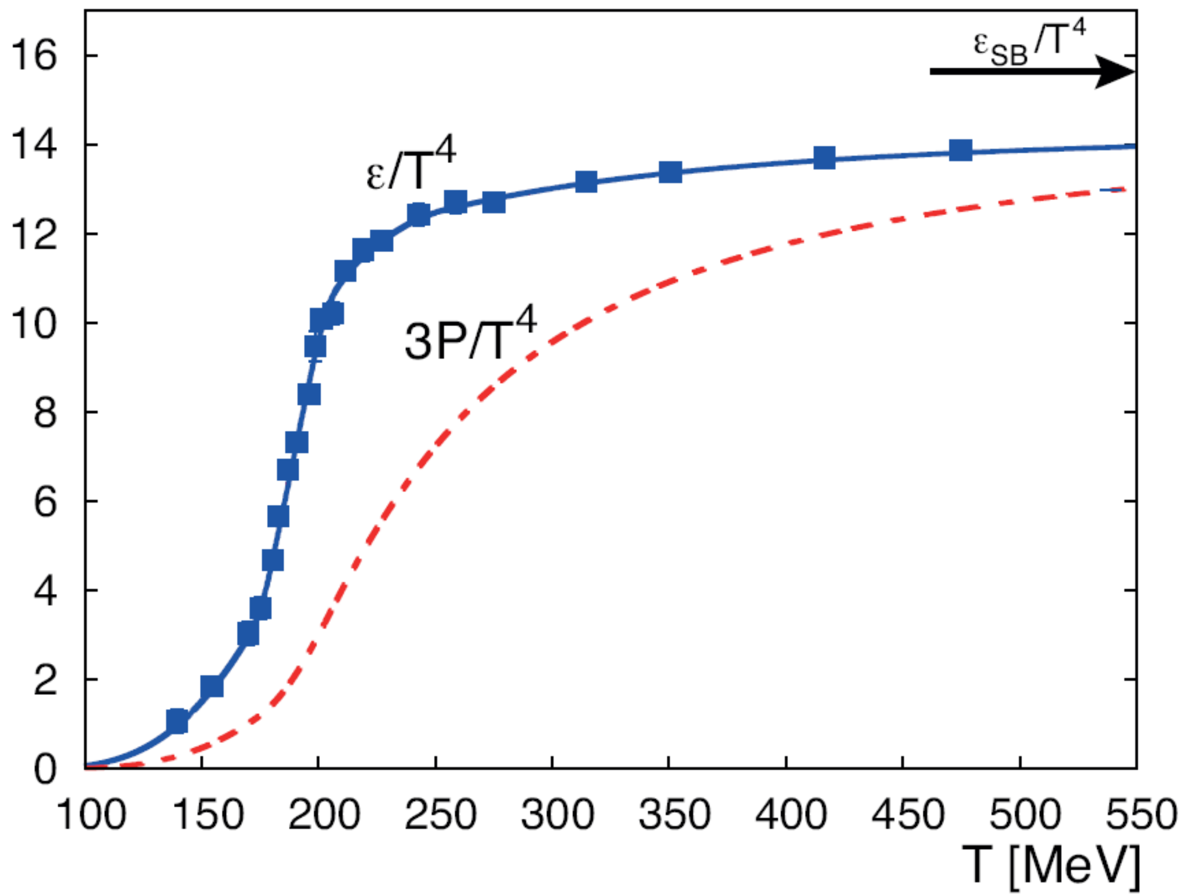
- This has provided a motivation to try to understand the dynamics of QGP through the **gauge/string duality**

Dual description of QGP as a part of Gauge/string duality

- There is not yet exist a gravity dual construction for QCD.
- Differences between N = 4 SYM and QCD are less significant, when quarks and gluons are in the deconfined phase (because of the conformal symmetry at the quantum level N = 4 SYM theory does not exhibit confinement.)
- Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures T >300 MeV and the equation of state can be approximated by E = 3 P (a traceless conformal energy-momentum tensor).
- This motivates to use the AdS/CFT correspondence as a tool to get non-perturbative dynamics of QGP.
- There is the considerable success in description of the static QGP.

Review: Solana, Liu, Mateos, Rajagopal, Wiedemann, 1101.0618

I.A., Holographic approach for quark-gluon plasma in heavy ion collsions, UFN, v184, 2014

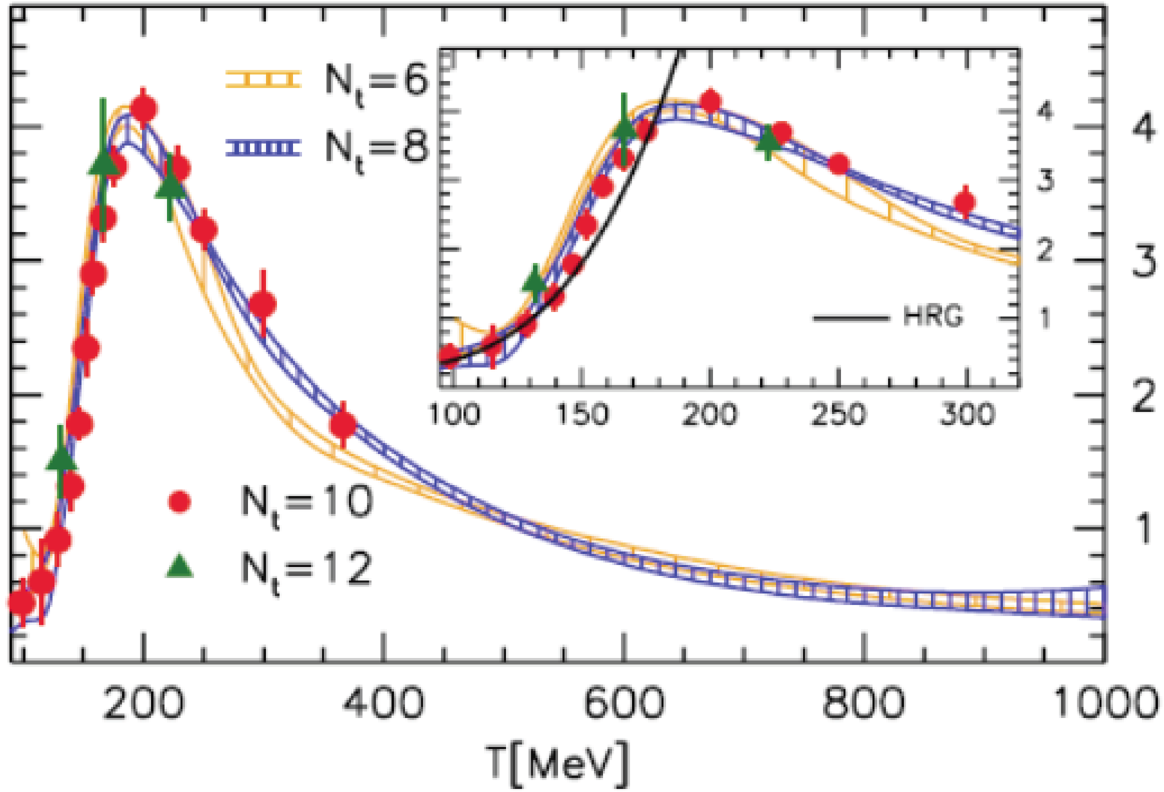


lattice calculation of QCD thermodynamics $N_f = 3$

S. Borsanyi et al., ”The QCD equation of state with dynamical quarks,” arXiv:1007.2580

Trace anomaly: (ε-3p)/T^4

[Borsanyi et al, 2010; Nf =2+1]



Quasi-conformal trend in a window of T > 2 Tc

Holography and AdS/CFT correspondence

$$< e^{\int_{\partial M} \phi_0 \mathcal{O}} > = e^{S_g[\phi_c(\phi_0)]}$$

Maldacena, 1997
Gubser,Klebanov,Polyakov
Witten, 1998

M=AdS, BHAdS,…

$$\phi(t,\vec{x},z), \quad S_g[\phi], \quad \delta S_g[\phi_c]=0$$

$$\phi_c \,|_{\partial M} = \phi_0$$

+ requirement of regularity at horizon

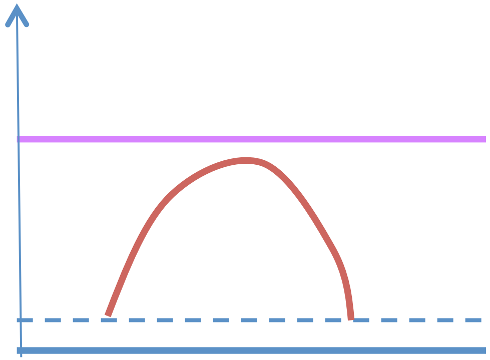
Correlators with/without Temperature via AdS/CFT

Example I. AdS, D=2+1

$$ds^2 = \frac{-dt^2 + dx^2 + dz^2}{z^2}$$

$$< O_\Delta(t,x) O_\Delta(t,x') > \sim \frac{1}{|x - x'|^{2\Delta}}$$

Example II. BHAdS, D=2+1



$$\mathrm{ds}^2 = \frac{1}{z^2}(f(z)dt^2 + \frac{dz^2}{f(z)} + d\vec{x}^2)$$

$$\mathrm{f(z) = 1 - Mz^2}$$

$$r_H = 2\pi T$$ Temperatute

$$< O_\Delta(t,x) O_\Delta(t,x') >_T \sim \frac{1}{|\sinh(\pi T |x - x'|)|^{2\Delta}}$$

Bose gas

Holography for thermal states

TQFT in

M_D-spacetime

=

Black hole

in AdS_{D+1}-space-time

TQFT = QFT with temperature

Hologhraphic Description of Formation of QGP

(Hologhraphic thermalization)

Thermalization of QFT in Minkowski D-dim space-time

↔

Black Hole formation in Anti de Sitter (D+1)-dim space-time

Models of BH creation in D=5 and their meaning in D=4

$$g_{MN} \Rightarrow g_{MN}{}^{(0)} + g_{MN}{}^{(1)}$$

- AdS/CFT correspondence

$$Z_{ren}(z_0)\ \ g_{\mu\nu}{}^{(1)}\,\Big|_{\substack{boundary \\ z_0 \to 0}} = T_{\mu\nu}$$

Main idea: make some perturbation of AdS metric that near the boundary “mimic” the mater (heavy ions) collisions and see what happens.

Hologhraphic thermalization

How to “mimic” the heavy ions collision

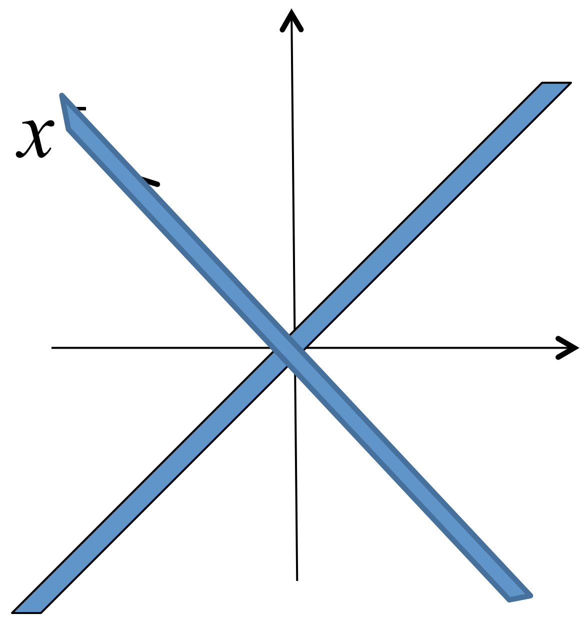
Models:

- shock waves collision in AdS
- colliding ultrarelativistic particles in AdS_3 (toy model)
- infalling shell

Nucleus collision in AdS/CFT

$$\langle T_{--} \rangle \sim \mu\, \delta(x^-)$$

$$\langle T_{++} \rangle \sim \mu\, \delta(x^+)$$

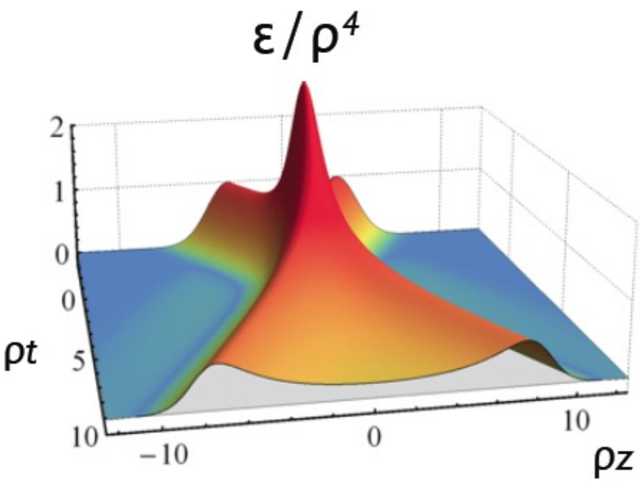


$$ds^2 = \frac{L^2}{z^2}\left[-2\,dx^+dx^- + \frac{2\pi^2}{N_C^2}\langle T_{--}(x^-)\rangle z^4\, dx^{-2} + + \frac{2\pi^2}{N_C^2}\langle T_{++}(x^+)\rangle z^4\, dx^{+2} + dx_\perp^2 + dz^2\right]$$

The metric of two shock waves in AdS corresponding to collision of two ultrarelativistic nucleus in 4D

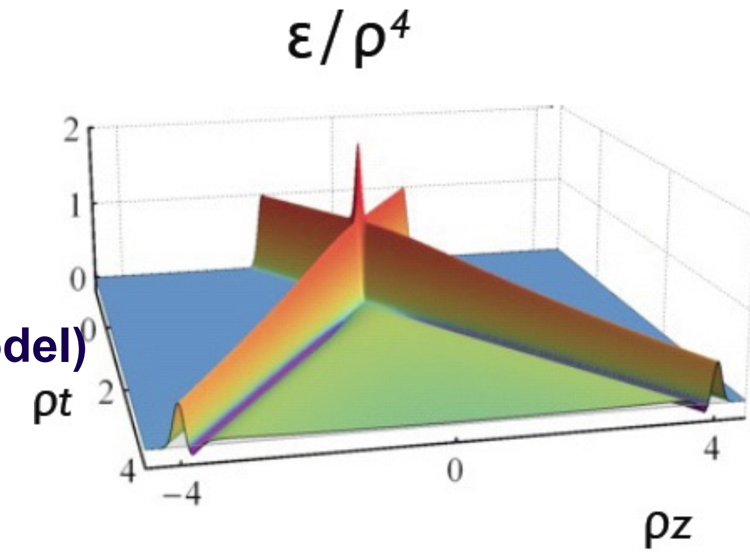
Holographic collision of two gaussian shocks

From *Chesler & Yaffe*



Low Energy Shocks

(toy model)



High energy shocks

Shocks pass through each other

Hologhraphic thermalization

Physical quantities that we expect to estimate:

D=5 AdS

- Black hole formation time
- Entropy

D=4 Minkowski

- Thermalization time
- Multiplicity

Thermalization time

Experimental data (just estimations)

$$\epsilon(y) = \frac{1}{A\tau_{therm}} \frac{N}{dy} < m_{tr} >, \quad m_{tr} = \sqrt{m_\pi^2 + k_{tr}}$$

Bjorken, 1983

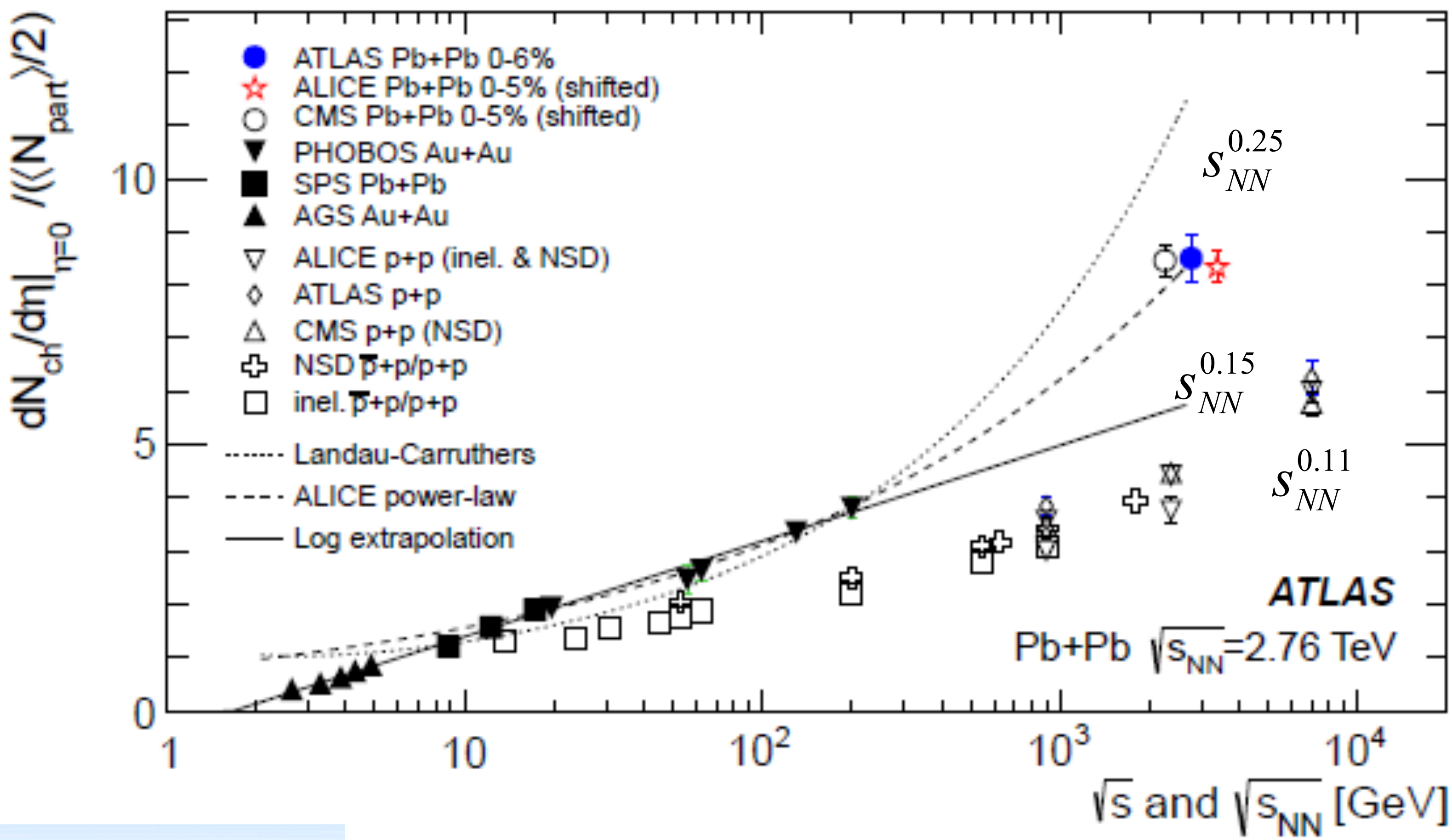
Holographic estimations

Position of horizon ~ size of the trapped surface

Multiplicity

Experimental data

Plot from: ATLAS Collaboration 1108.6027



PbPb

$$dN_{ch}/d\eta \propto s_{NN}^{0.15}$$

pp:

$$dN_{ch}/d\eta \propto s_{NN}^{0.11}$$

Multiplicity as entropy

D=4. Macroscopic theory of high-energy collisions

Landau(1953); Fermi(1950)

thermodynamics, hydrodynamics, kinetic theory, …

D=5. Holographic approach

Main conjecture: multiplicity is proportional to entropy of produced D=5 Black Hole

$$\mathcal{M} \sim S$$

Gubser et al: 0805.1551

The mininal black hole entropy can be estimated by trapped surface area

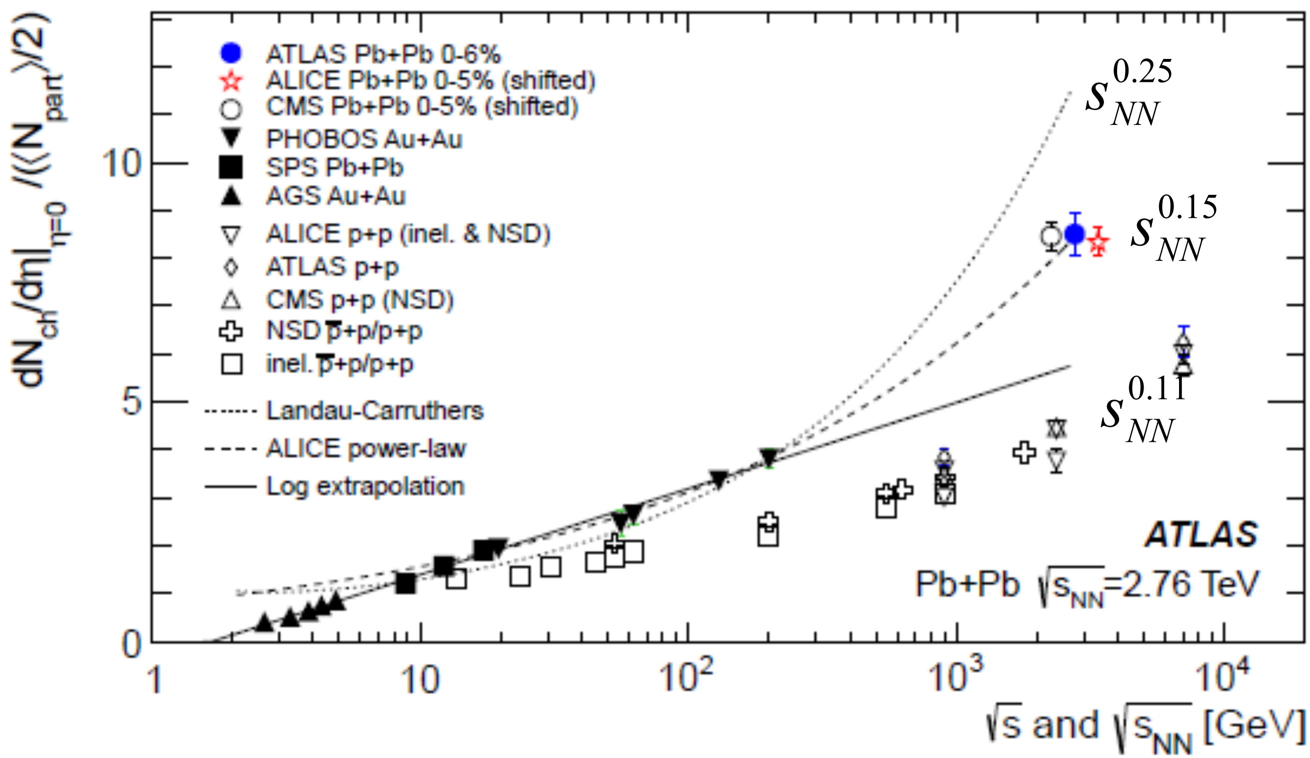
$$S \geq S_{trapped} = A_{trapped} / 4G_N$$

Gubser, Pufu, Yarom, JHEP , 2009
Alvarez-Gaume, C. Gomez, Vera,
Tavanfar, Vazquez-Mozo, PLB, 2009
IA, Bagrov, Guseva, JHEP, 2009
Kiritsis, Taliotis, JHEP, 2011

Multiplicity: Hologhrapic formula vs experimental data

The simple holographic model gives

$$dN_{ch}/d\eta \sim s_{NN}^{1/3}$$



Search for models with suitable entropy

Metric with modified b-factor **IHQCD** **Gursoy, Kiritsis, Nitti**

$$S_5 = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{4}{3}(\partial\Phi)^2 + V(\Phi_s)\right] dx^5$$

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2)$$

$$\alpha = e^{\Phi} \qquad \beta = b\frac{d\alpha}{db} \qquad \beta(\alpha) = -b_0\alpha^2 - b_1\alpha^3$$

$$V(\alpha) = \frac{12}{L^2}\left\{1 + V_0\alpha + V_1\alpha^{4/3}\left[\log\left(1 + V_2\alpha^{4/3} + V_3\alpha^2\right)\right]^{1/2}\right\}$$

Reproduces 2-loops QCD beta-function

Reproduce an asymptotically-linear glueball spectrum

Search for models with suitable entropy

Kiritsis, Taliotis, **JHEP(2012)**

Shock wave metric with modified b-factor

$$ds^2 = b^2(z)\left(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi(z,x^1,x^2)\delta(x^+)(dx^+)^2\right)$$

The Einstein equation for particle in dilaton field

$$\left(R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R\right) - \frac{g_{\mu\nu}}{2}\left(-\frac{4}{3}(\partial\Phi_s)^2 + V(\Phi_s)\right) - \frac{4}{3}\partial_\mu\Phi_s\,\partial_\nu\Phi_s - g_{\mu\nu}\frac{d(d-1)}{2L^2} = 8\pi G_5 J_{\mu\nu}$$

Typical behavour

$$b(z) = \frac{L}{z}e^{-z^2/z_0^2}$$

$$s_{NN}^{\delta_1}\ln^{\delta_2} s_{NN}$$

$$\delta_1 \approx 0.225, \quad \delta_2 \approx 0.718$$

Shock wall with modified by b-factor

Description of HIC by the wall-wall shock wave collisions

S. Lin, E. Shuryak, 0902.1508
I. A., Bagrov and E.Pozdeeva, JHEP(2012)

$$ds^2 = b^2(z)\left(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi^w(z, x^1, x^2)\delta(x^+)(dx^+)^2\right)$$

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^w(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z)$$

I. A., E.Pozdeeva,T.Pozdeeva (2013,2014)

$s_{points} \sim s_{walls}$

Power-law b-factor

$$b = (L/z)^a$$

$$S_{walls} = \frac{L}{2G_5}\left(\frac{8\pi G_5}{L^2}\right)^{\frac{3a-1}{3a}} E^{\frac{3a-1}{3a}}$$

The multiplicity depends as $s^{0.15}_{NN}$ in the range $10\text{-}10^3$ GeV

Power-law b-factor coinsides with experimental data at *a*≈0.47.

We consider

$$b(z) = \frac{L}{z^{1/2}}$$

Price: non standard kinetic term!

Multiplicity with anisotropic Lifshitz background

IA, A. Golubtsova

$$S=\frac{1}{2\kappa^2}\int d^5x\sqrt{|g|}\left[R-2\Lambda-\frac{1}{12}H_3^2-\frac{m_0^2}{2}B_2^2\right]$$

$$H_3=2\sqrt{\frac{\nu-1}{\nu}}\rho d\rho\wedge dt\wedge dx,\quad B_2=\sqrt{\frac{\nu-1}{\nu}}\rho^2 dt\wedge dx$$

$$\Lambda=5+\frac{6}{\nu}+\frac{3}{\nu^2}$$

$$ds^2=\rho^2\left(-dt^2+dx^2\right)+\rho^{2/\nu}\left(dy_1^2+dy_2^2\right)+\frac{d\rho^2}{\rho^2}$$

Shock wave

$$z=1/\rho$$

$$ds^2=\frac{\phi(y_1,y_2,z)\delta(u)}{z^2}du^2-\frac{1}{z^2}dudv+z^{-2/\nu}\left(dy_1^2+dy_2^2\right)+\frac{dz^2}{z^2}$$

Solves E.O.M. if

$$\Box_3$$

$$\delta(u)\left[\Box_3-\left(1+\frac{2}{\nu}\right)\right]\frac{\phi(y_1,y_2,z)}{z}=-2zT_{uu}$$

$$ds^2=\rho^{2/\nu}\left(dy_1^2+dy_2^2\right)+\frac{d\rho^2}{\rho^2}$$

Multiplicity with anisotropic Lifshitz background

Domain wall

$$\left[\Box_3 - \left(1+\frac{2}{\nu}\right)\right]\frac{\phi(z)}{z} = -16\pi G_5 z J_{uu} \qquad J_{uu} = z^{1+2/\nu}\delta(z-z_0)$$

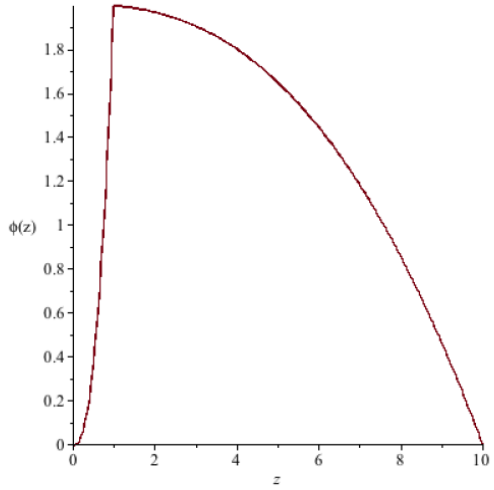
$$\phi(z) = -\frac{8\nu\pi G_5 E}{\nu+1} z_0^{2(\nu+1)/\nu}\Theta(z-z_0)\left(\frac{z^{2(\nu+1)/\nu}}{z_0^{2(\nu+1)/\nu}} - 1\right) + C_1 z^{2(\nu+1)/\nu} + C_2$$

Multiplicity with anisotropic Lifshitz background

Colliding Domain Walls

$$ds^2 = -\frac{1}{z^2}dudv + \frac{1}{z^2}\phi_1(y_1, y_2, z)\delta(u)du^2 + \frac{1}{z^2}\phi_2(y_1, y_2, z)\delta(u)dv^2$$

$$+ \frac{1}{z^{2/\nu}}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{z^2}$$



S~ $\frac{\nu}{4G_5}(8\pi G_5)^{2/(\nu+2)}E^{2/(\nu+2)}$

$$3a = 1 + \frac{2}{\nu}$$

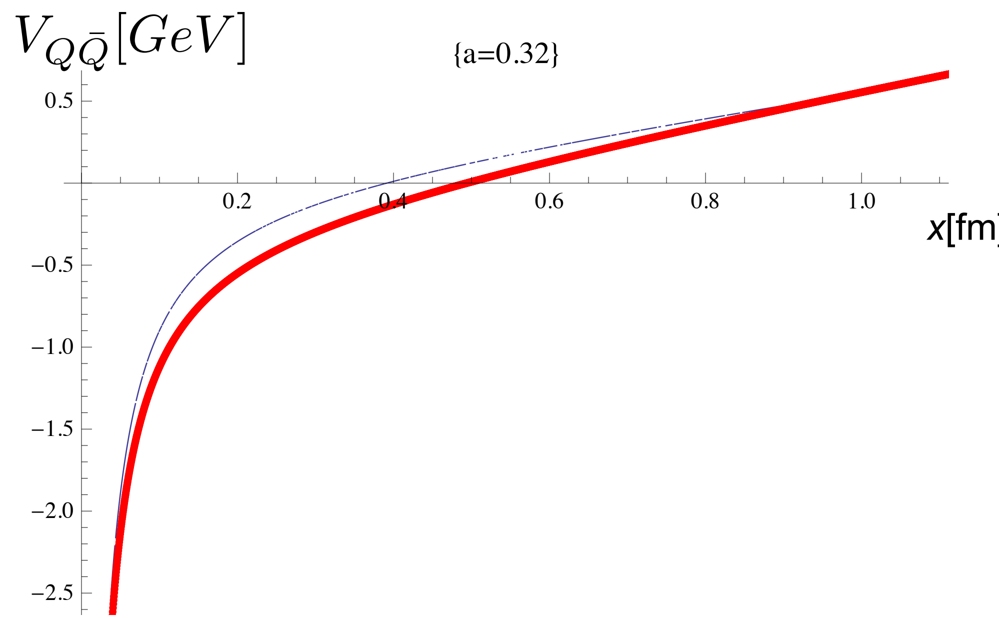
Multiplicity and quark potential

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2)$$

$$b^2(z) = \frac{L^2 h(z)}{z^2}$$

$$h = e^{\frac{a z^2}{2}}$$

AdS with soft-wall



$$V_{Cornell}(x) \equiv V_{Q\bar{Q}}(x) = -\frac{\kappa}{x} + \sigma_{str} x + V_0$$

$$\kappa \approx 0.48, \quad \sigma_{str} = 0.183 GeV^2, \quad C = -0.25 GeV$$

Coulomb term

Confinement linear potential

O. Andreev and V. Zakharov
hep-ph/0604204

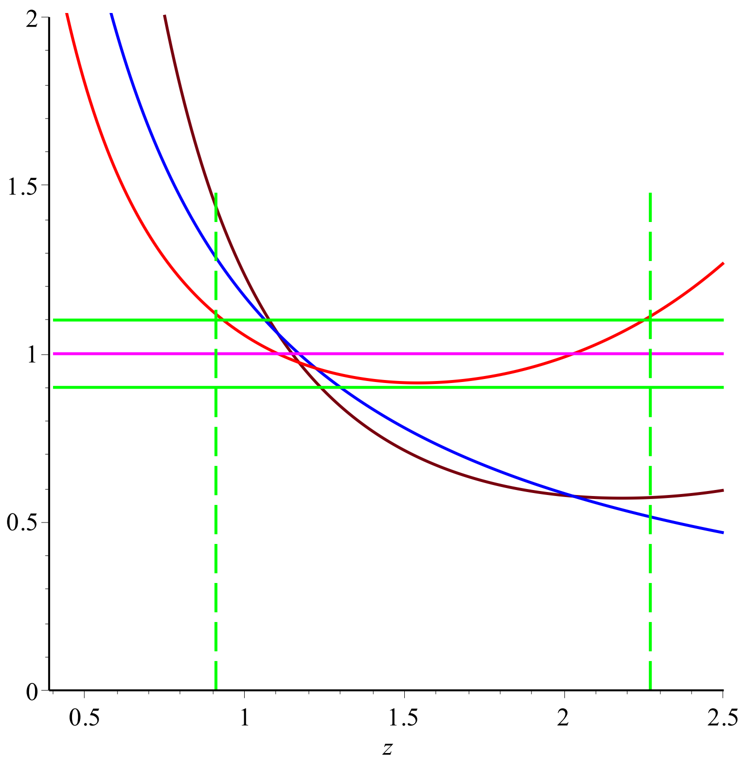
Multiplicity and quark potential

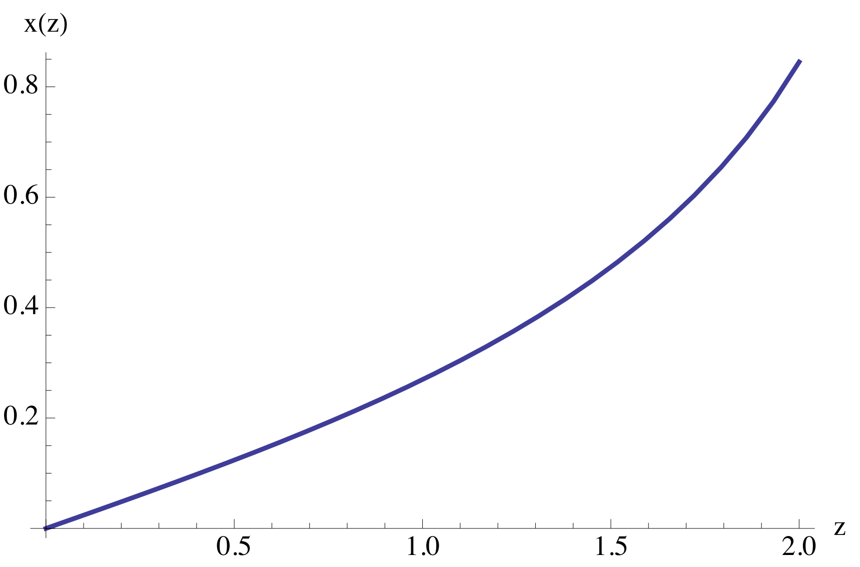
with D.Ageev

$$\frac{L^2 e^{\frac{az^2}{2}}}{z^2} \approx \frac{L^2}{z L_{eff}}$$

$$z_{UV} < z < z_{IR}$$

$$L_{eff} = 0.95$$





Multiplicity and quark potential

Quark potential in string frame

$$ds_s^2 = e^{2\mathcal{A}(z)}(-dt^2 + d\vec{x}^2 + dz^2)$$

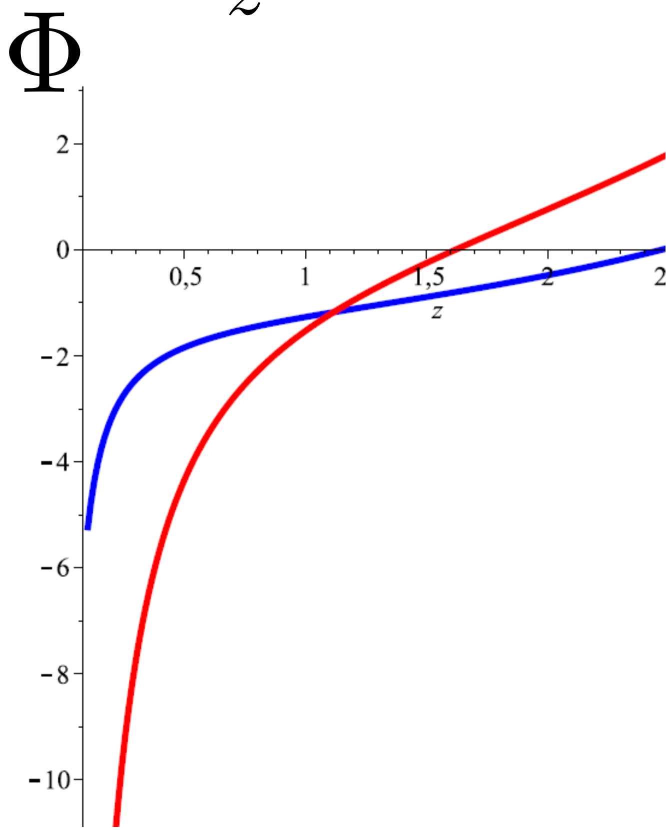
$$\frac{L^2 e^{\frac{a z^2}{2}}}{z^2} = e^{2\mathcal{A}}$$

Trapped surface in Einstein frame

$$ds_E^2 = e^{2A(z)}(-dt^2 + d\vec{x}^2 + dz^2)$$

$$A(z) = -\frac{2}{3}\Phi + \mathcal{A}$$

$$\Phi'' - 2\mathcal{A}'\Phi' = \frac{3}{2}(\mathcal{A}'' - \mathcal{A}'^2)$$



Multiplicity and quark potential

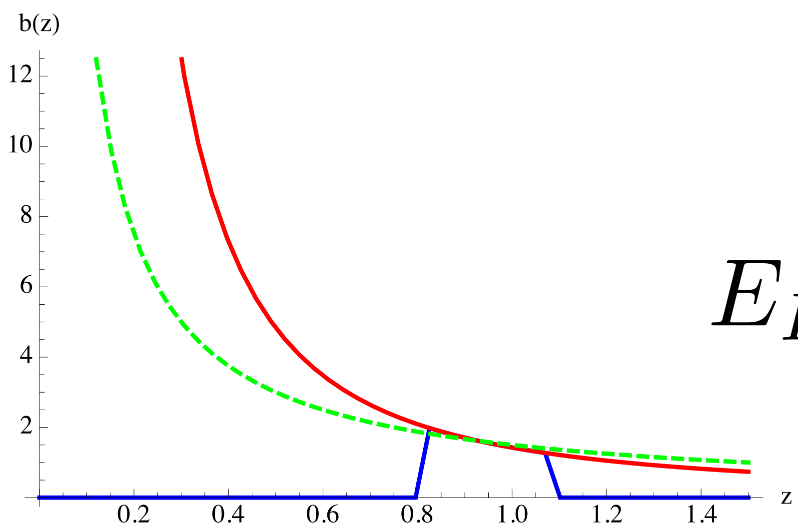
$$\frac{e^{-\frac{3}{4}\Phi+\frac{a z^2}{2}}}{z^2} \approx \frac{L_{eff}}{z}$$

Trapped surface

$$z_{UV} < z < z_{IR}$$

$$z_a < z < z_b$$

Pack the trapped surface in the interval $z_{UV} < z < z_{IR}$



$$E_{IR} < E < E_{UV}$$

$$E_{IR,UV} = \frac{L_{eff}^{7/2}}{8\pi G_5} \cdot \frac{1}{z_{IR,UV}^{3/2}}$$

Estimation of the termalization time!

$$s \sim (L_{eff} E)^{1/3}$$

Conclusion

Formation of QGP of 4-dim QCD ⇔ Black Hole formation in AdS_5

- **b-factor that fits experimental data:**

1) Multiplicity

$$S_{data} \propto s_{NN}^{0.15}$$

2)Cornell qq-potential

$$V_{qq}(x) = -\frac{\kappa}{x} + \sigma_{str} x + V_0$$

- **attempts to get top-down model for this b-factor**