Time-Machine Deformation of Conformal Correlation Functions

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Plan

- I. Introduction. (Reminder from previous talk: Holographic Models of QGP formation)
- A. Shock waves collisions in AdS5
- B. Infalling shell
- C. 3-dim toy model
- II. 3 -dim toy models

  Ultrarelativistic particles

  Time-machine

with A.Bagrov

Introduction. B. Infalling shell as holographic model of thermalization

d+1-dimensional infalling shell geometry is described in Poincar'e coordinates by the Vaidya metric

Danielsson, Keski-Vakkuri and Kruczenski

$$ds^2 = \frac{1}{z^2}\left[-\left(1 - m(v)z^d\right)dv^2 - 2dz\,dv + d\mathbf{x}^2\right] \qquad ★$$

v labels ingoing null trajectories

1) For constant $m(v) = M$, the coordinate transformation $dv = dt - \frac{dz}{1 - M\,z^d}$

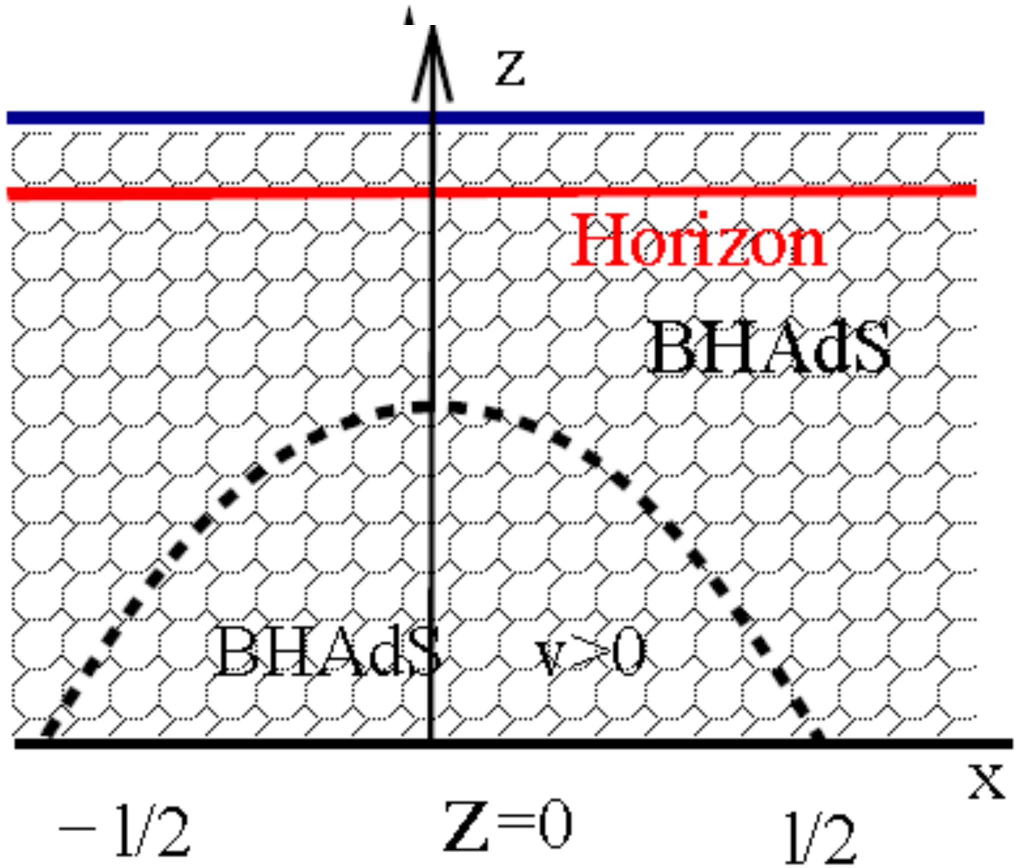
brings ★ in the form

$$ds^2 = \frac{1}{z^2}\left[-\left(1 - Mz^d\right)dt^2 + \frac{dz^2}{1 - Mz^d} + d\mathbf{x}^2\right]$$

2) $$m(v) = \frac{M}{2}\left(1 + \tanh\frac{v}{v_0}\right)$$

Thermalization time

Thermalization with Vadya AdS



Introduction. B. Infalling shell as holographic model of thermalization

- **Thermalization time dependence on**
  - **chemical potential (ch.pot. decreases thermalization time ?)**
  - **anizotropy**
  - **non-centricity**

Centrality independence of thermalization time in Vaidya approximation (3D)

IA, Bagrov,Koshelev, JHEP (2013)

Question: can trust to **infalling shell estimations**

Model example: colliding particles in AdS_3

(main part of the talk)

Stationary Particles in Ad_3

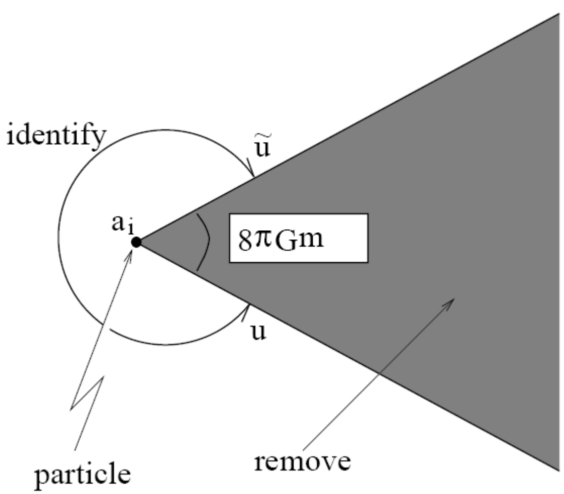
M $^{(1,2)}$

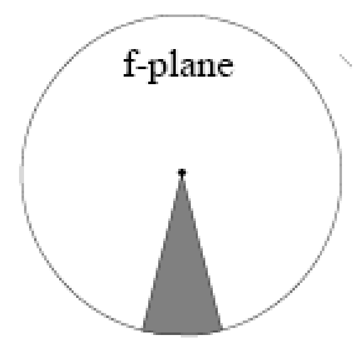
'**t Hooft, Deser, Jackiw, 1983**

$$G_{00} = \frac{8\pi m G}{\sqrt{g}}\delta(\vec{x}), \quad \vec{x} = (x^1, x^2),$$

$$G_{0i} = 0, \quad G_{ik} = 0, \quad i,k = 1,2$$

$$ds^2 = -dt^2 + \frac{1}{|\vec{x}|^{8mG}}(dx_1^2 + dx_2^2)$$





Particles are described by the angle deficit

AdS_3 in Global Coordinates

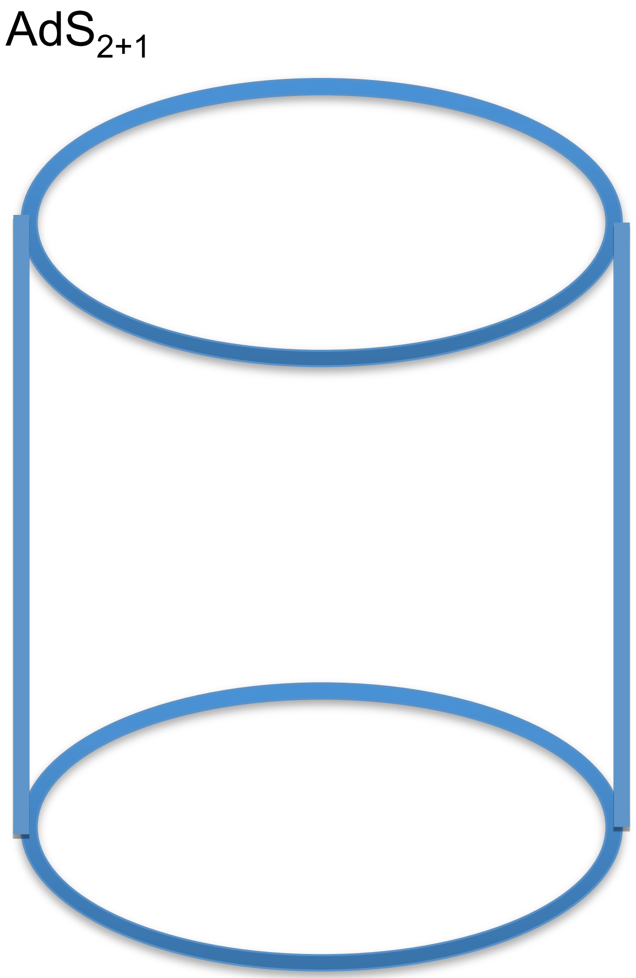
$$AdS_3 \sim SL(2,R)$$

$$\mathbf{x} = \frac{1+r^2}{1-r^2}\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} + \frac{2r}{1-r^2}\begin{pmatrix} \sin\phi & \cos\phi \\ \cos\phi & -\sin\phi \end{pmatrix}$$

$$ds^2_{AdS_{2+1}} = \frac{1}{2}\mathrm{Tr}(\mathrm{x}^{-1}d\,\mathrm{x}\,\mathrm{x}^{-1}d\,\mathrm{x})$$

$$ds^2_{AdS_{2+1}} = R^2\left[-\left(\frac{1+r^2}{1-r^2}\right)^2 dt^2 + \left(\frac{2}{1-r^2}\right)^2 (dr^2 + r^2 d\phi^2)\right]$$

$$ds^2_{AdS_{2+1}} = R^2 \left[-\left(\frac{1+r^2}{1-r^2}\right)^2 dt^2 + \left(\frac{2}{1-r^2}\right)^2 (dr^2 + r^2 d\phi^2) \right]$$

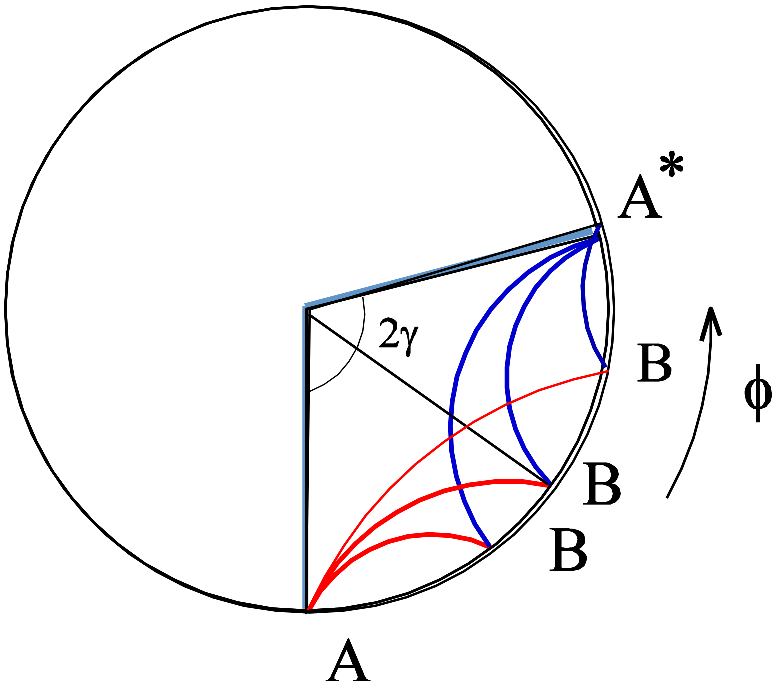


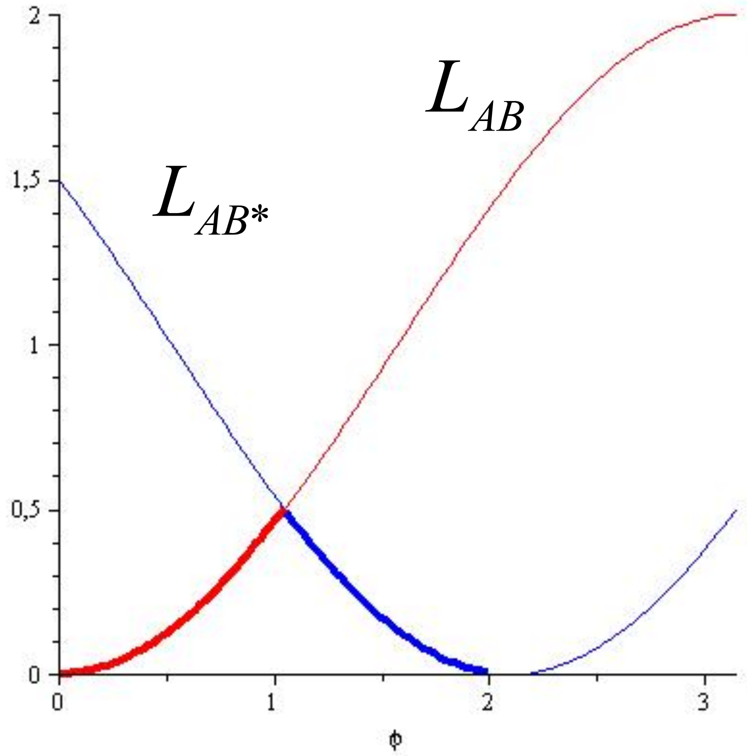
2-point correlation function

Correlators for Particles in AdS$_3$

Section of cylinder with a particle

$$l_{AB} = \ln(L_{AB})$$

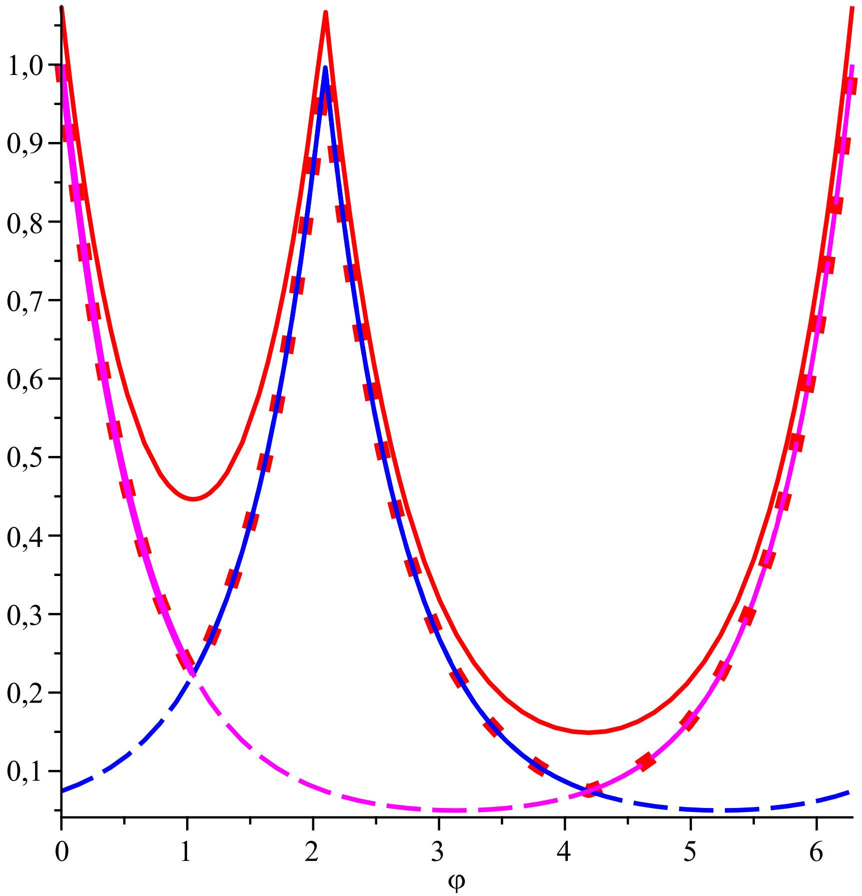




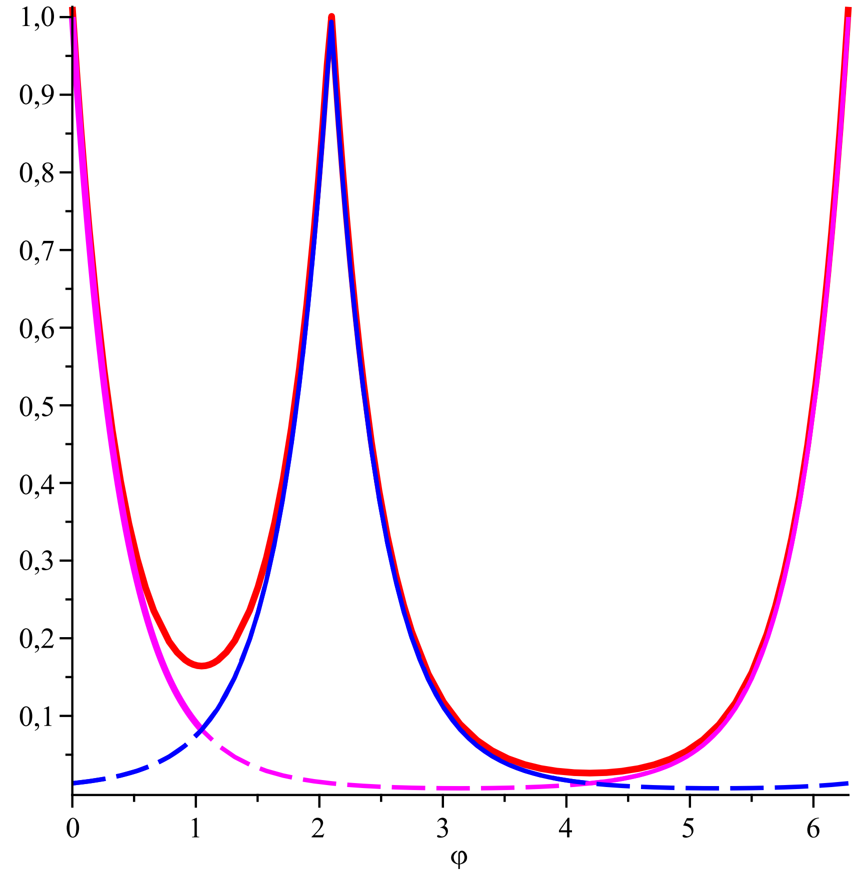
$$< O_\Delta(A) O_\Delta(B) > \sim e^{-\Delta l_{\min}}$$

ANSWER!

Two geodesic approximation for equal time correlation functions



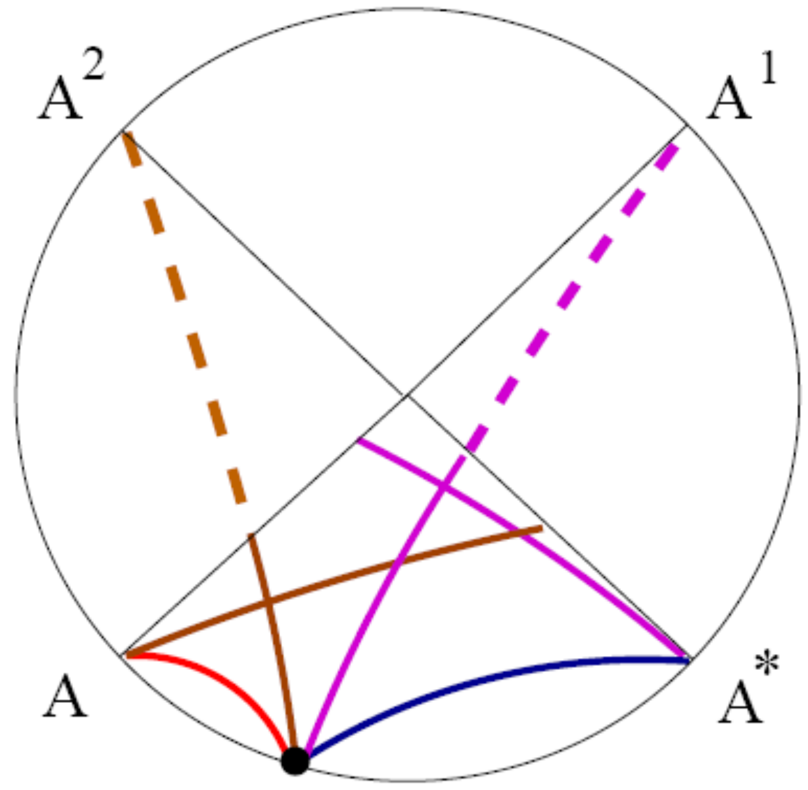
$$\Delta = 1$$



$$\Delta = 3$$

2-point correlation functions on $S^1 x R^1$ dual to AdS_3 with stationary particle at the center

Geodesics with winding



2-point correlation functions on S^1xR^1 dual to AdS_3 with stationary particle at the center

$$\frac{1}{L_0}\int_0^{L_0} d\phi \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itp-i\phi\frac{2\pi}{L_0}r} < \Phi_0(t,\phi)\Phi_0(0,0) >_{1-part.\, in\ center}$$

$$||$$

$$< \Phi_0(t,\phi)\Phi_0(0,0) >_L \theta(\mathcal{L}(0,\phi) - \mathcal{L}(\phi,\phi_0))$$

$$+ < \Phi_0(0,L_0)\Phi_0(t,\phi) >_L \theta(-\mathcal{L}(0,\phi) + \mathcal{L}(\phi,\phi_0))$$

The angle deficit = π

$$G(p,r,\pi) = \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{(2n+1)(-1)^n}{p^2+(2n+1)^2} \cdot \frac{(-1)^r}{(2n+1)^2-4r^2}$$

Ultrarelativistic particles

$$\mathbf{x} = \frac{1+r^2}{1-r^2}\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} + \frac{2r}{1-r^2}\begin{pmatrix} \sin\phi & \cos\phi \\ \cos\phi & -\sin\phi \end{pmatrix}$$

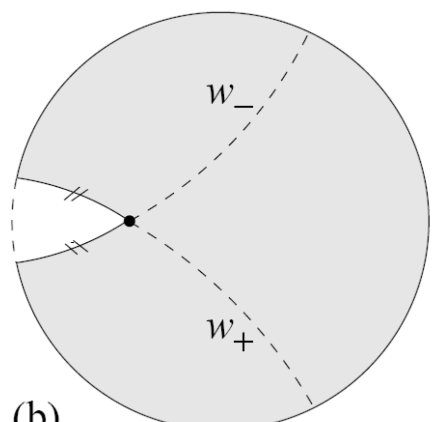
$$\mathbf{x} \to \mathbf{x}' = u^{-1}\mathbf{x}\,u \qquad u = 1 + \tan\epsilon \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

On the boundary

$$\tan t^* = \tan t(1+4\tan^2\epsilon) + 4\tan\epsilon\,\frac{\sin\phi}{\cos t} - 4\tan^2\epsilon\,\frac{\cos\phi}{\cos t}$$

$$\tan\phi^* = \frac{\sin\phi + 2\tan\epsilon(\sin t - \cos\phi)}{\cos\phi + 4\tan^2\epsilon\sin t + 4\tan\epsilon\sin\phi - 4\tan^2\epsilon\cos\phi}$$

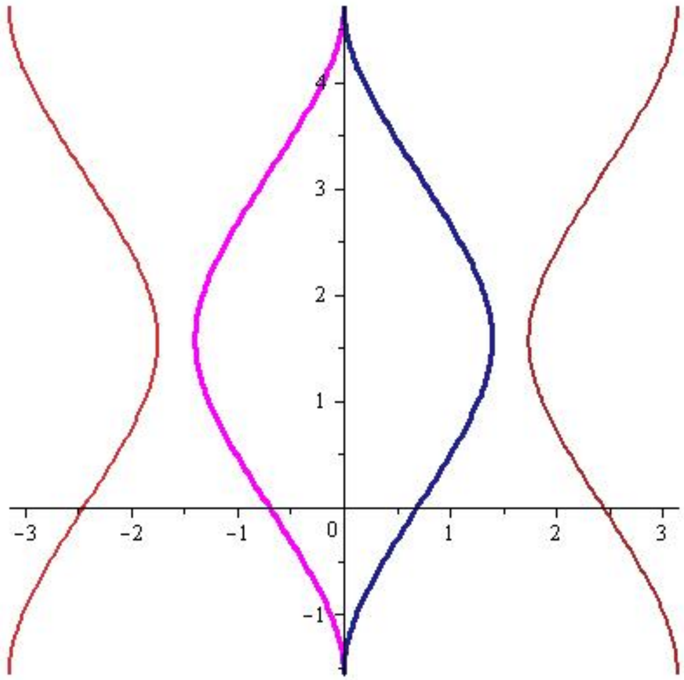
t=t* ⇒ $\sin t \sin\epsilon = \sin(\epsilon - \phi)$

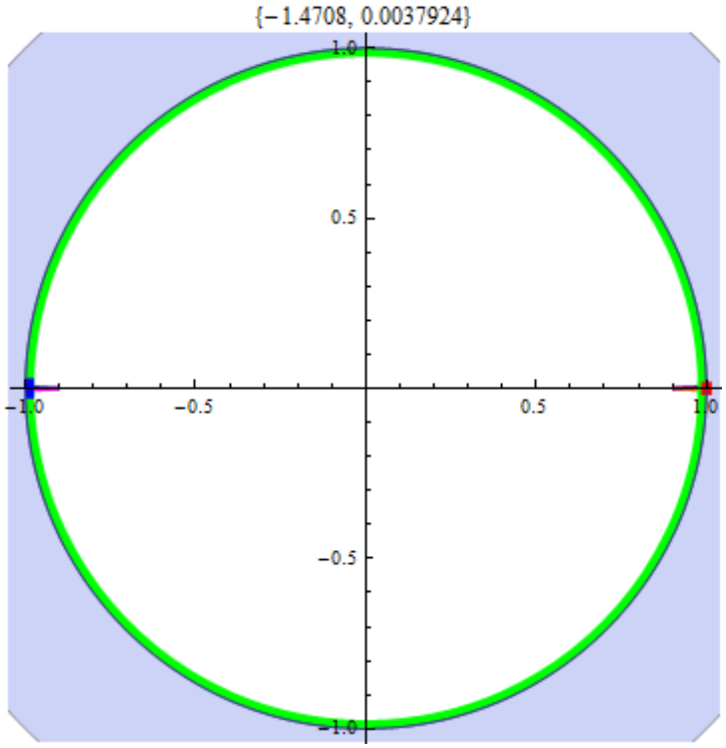


(b)

Matschull,9809087

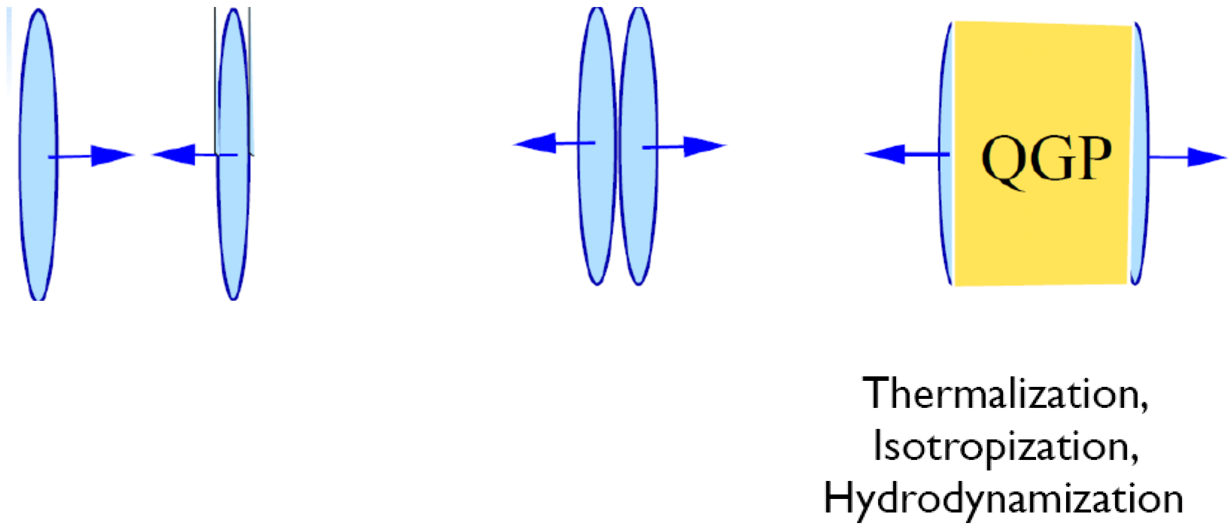
Two particles





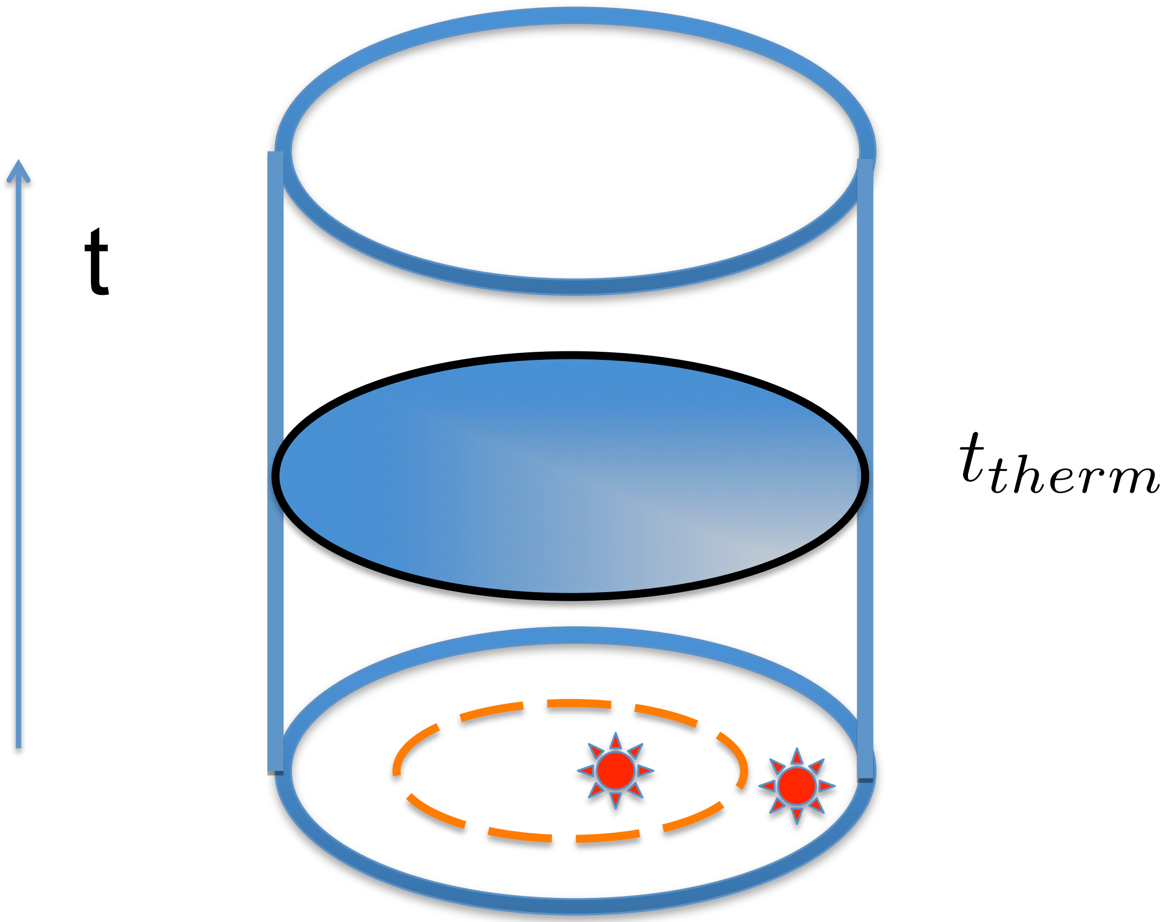
Hourglass

n>2 container with flexible walls



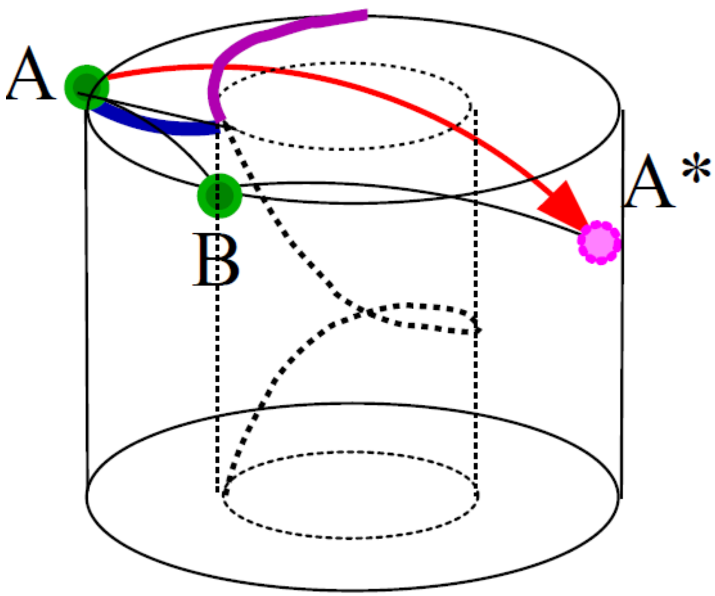
Container with flexible wall

Appearance of the trapped surface



2-point functions dual to AdS$_3$ with a massive moving particles

Particle moving along the spiral



Moving particle (along a spiral)

$$\mathbf{y} = \cosh\chi\,\mathbf{\Omega}(t) + \sinh\chi\,\mathbf{\Gamma}(\phi) \quad \longrightarrow \quad \mathbf{y}' = \mathbf{\Omega}(\frac{\alpha}{2})\cdot\mathbf{y}\cdot\mathbf{\Omega}_{\mathbf{L}}(-\frac{\alpha}{2},\psi_0)$$

$$\mathbf{\Omega}(t) = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} \qquad \mathbf{\Gamma}(\phi) = \begin{pmatrix} \sin\phi & \cos\phi \\ \cos\phi & -\sin\phi \end{pmatrix}$$

$$\mathbf{\Omega}_{\mathbf{L}}(-\frac{\alpha}{2},\psi_0) \equiv \mathbf{L}^{-1}(\psi_0)\cdot\mathbf{\Omega}(-\frac{\alpha}{2})\mathbf{L}(\psi_0)$$

$$\mathbf{L}(\psi_0) = \cosh\psi_0\,\mathbf{\Omega}(0) + \sinh\psi_0\,\mathbf{\Gamma}(0)$$

$$\begin{aligned}
\text{Axis}: &\quad \mathbf{x}_0(\gamma) = \mathbf{\Omega}(\gamma), \quad \gamma - parameter \\
\text{Spiral}: &\quad \mathbf{x}_{spiral}(\gamma,\psi_0) = \cosh\psi_0\,\mathbf{\Omega}(\gamma) + \sinh\psi_0\,\mathbf{\Gamma}(\gamma) \\
&\qquad\qquad \gamma - parameter, \quad \psi_0 - fixed \\
\text{Relation}: &\quad \mathbf{x}_{spiral}(\gamma,\psi_0) = \mathbf{x}_0(\gamma)\mathbf{L}(\psi_0)
\end{aligned}$$

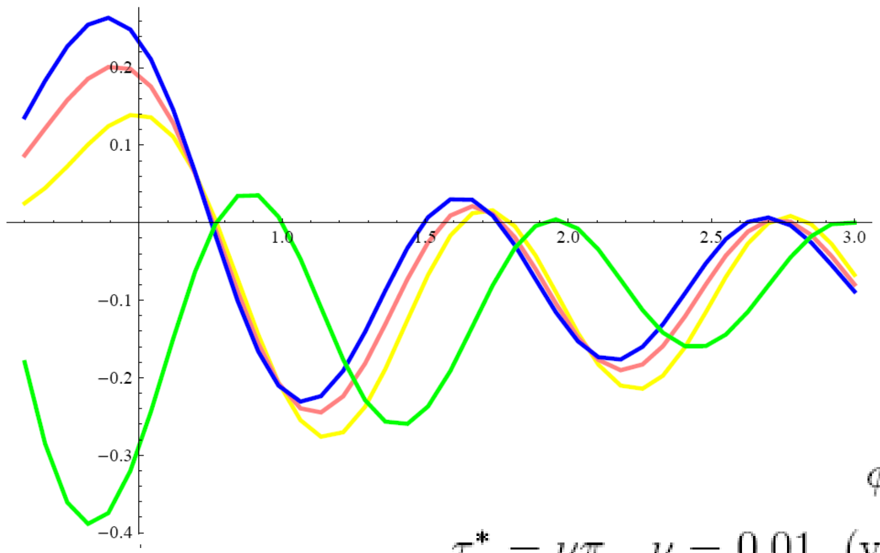
$$\begin{aligned}
\Delta\tau &= -2\arctan\left(\frac{\tanh\psi_0\sin\frac{\alpha}{2}}{1+\tanh\psi_0\cos\frac{\alpha}{2}}\right) \\
\Delta\phi &= -2\arctan\left(\frac{\sin\frac{\alpha}{2}}{\tanh\psi_0+\cos\frac{\alpha}{2}}\right)
\end{aligned}$$

2-point correlation functions on S^1xR^1 dual to AdS_3 with moving particle

$$G(p,r,\varphi^*,t^*) = \frac{1}{L_0}\int_0^{L_0} d\phi \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itp-i\phi\frac{2\pi}{L_0}r} < \Phi_0(t,\phi)\Phi_0(0,0) >_{1-part.\,moving\,along\,spiral}$$

$$||$$

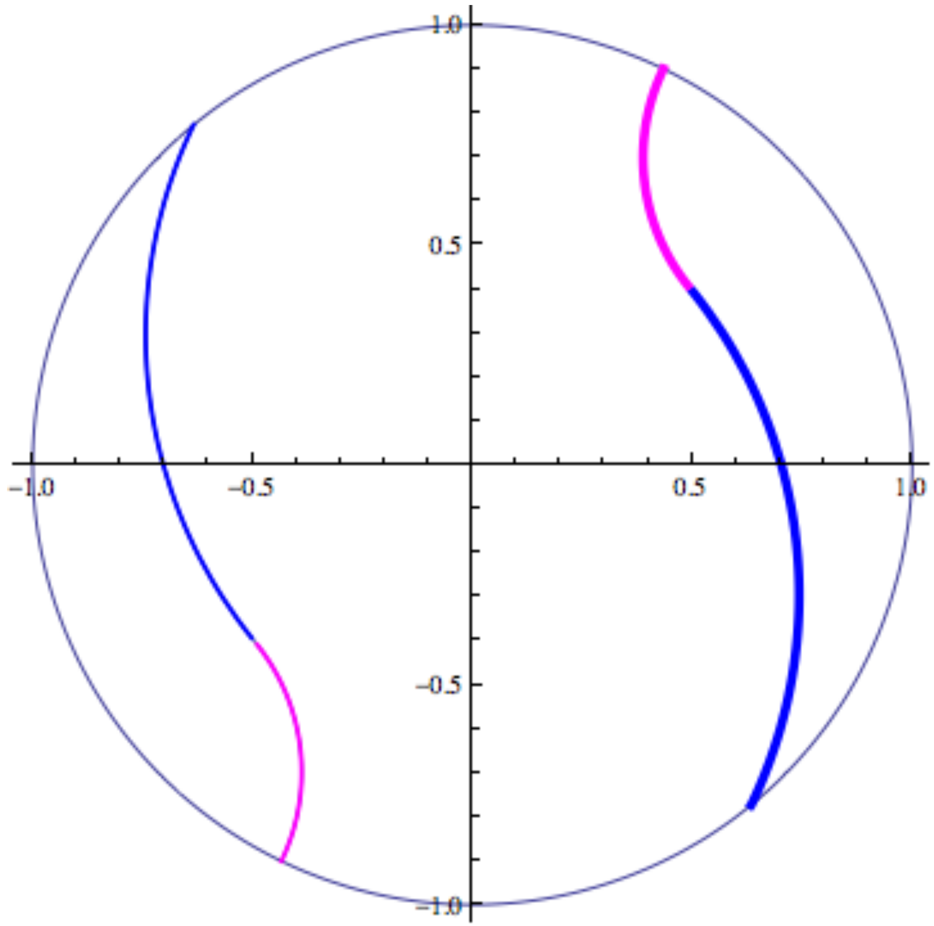
$$= \quad < \Phi_0(t,\phi)\Phi_0(0,0) >_L \theta(\mathcal{L}(t^*,\phi^*|\phi,t) - \mathcal{L}(\phi,t|0,0))$$
$$+ \quad < \Phi_0(t^*,\phi^*)\Phi_0(t,\phi) >_L \theta(\mathcal{L}(t^*,\phi^*|\phi,t) - \mathcal{L}(t^*,\phi^*|\phi,t)$$



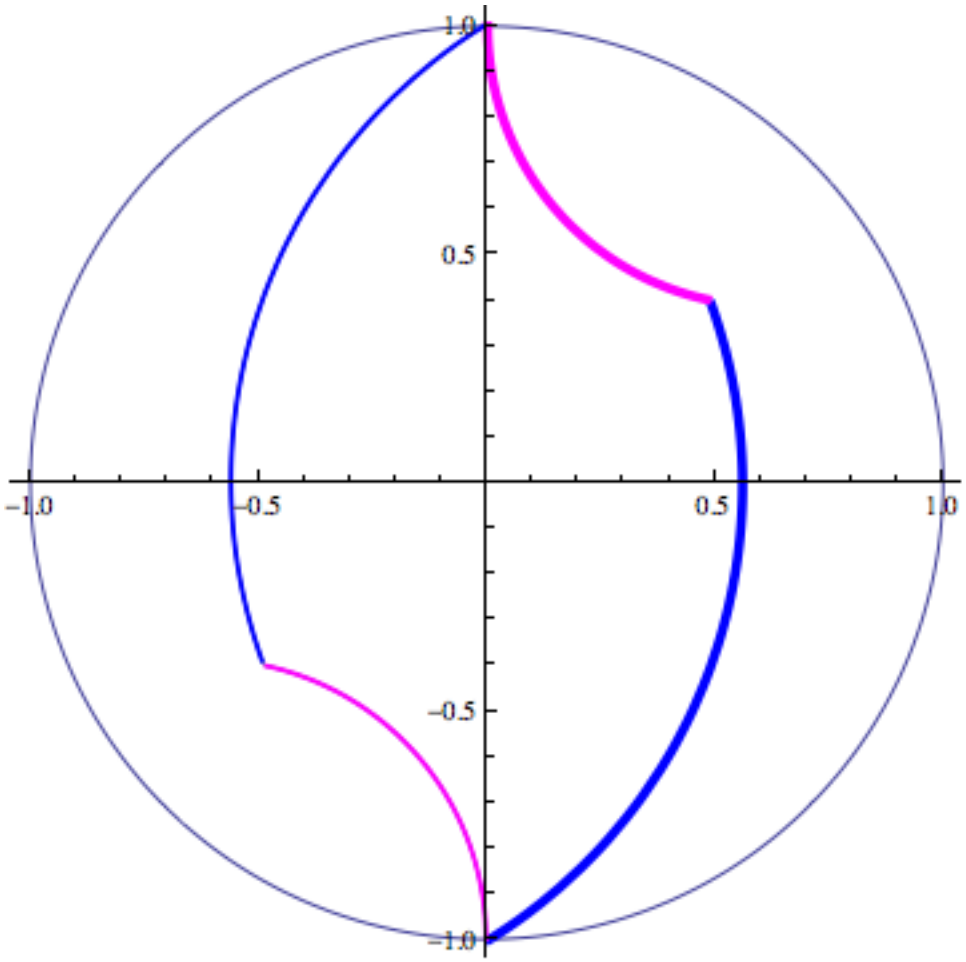
$\phi^* = 3\pi/2$

$\tau^* = \nu\pi$, $\nu = 0.01$, (yellow), $\nu = 0.05$ (pink), $\nu = 0.1$ (blue)

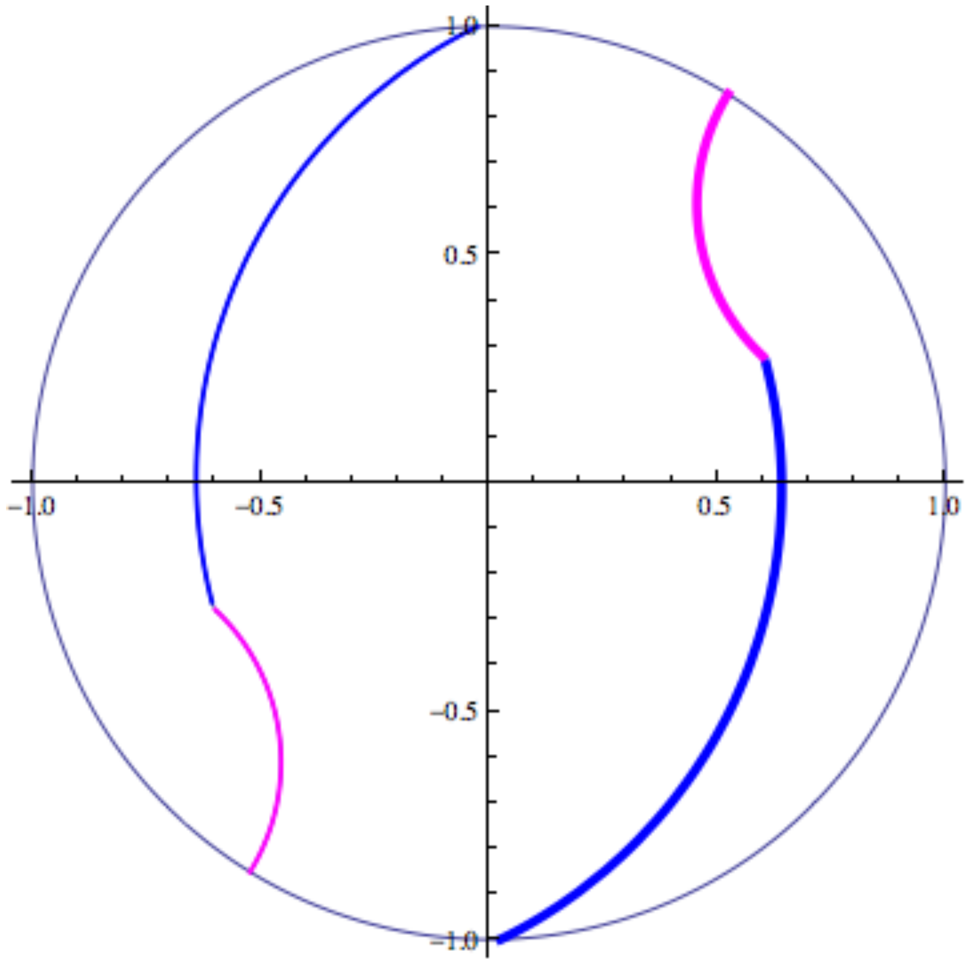
Two particles. Increasing angle (mass)



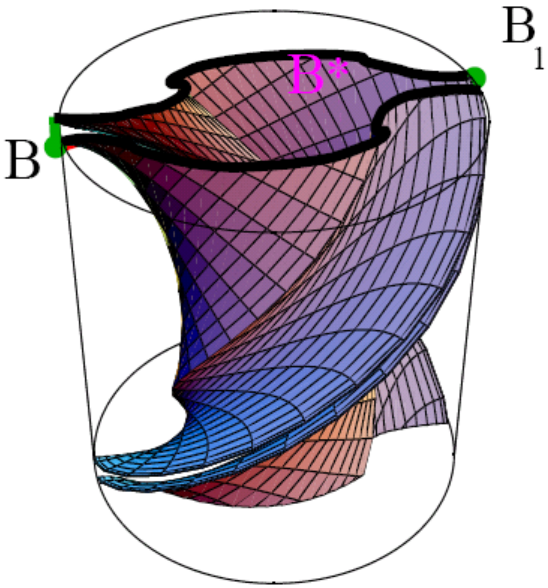
Two particles. Increasing Lorentz parameter.

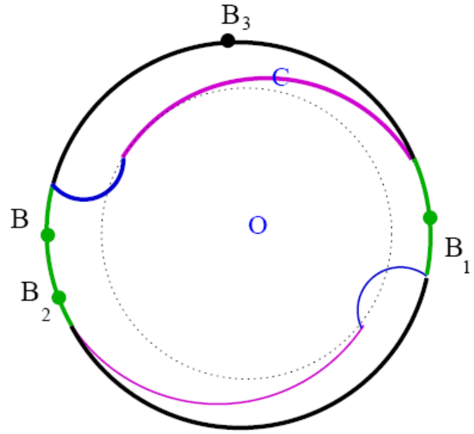


Time evolution



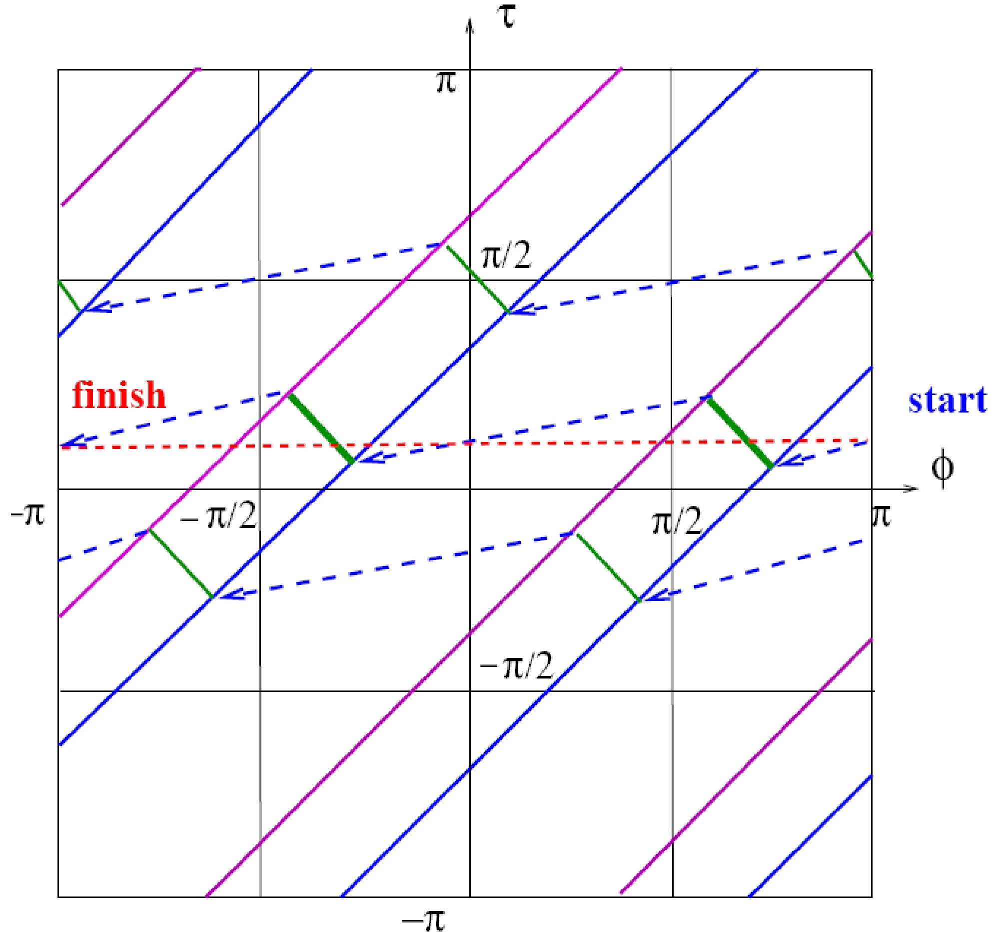
Gott's time machine





TM will be produced by two idenitical particles when the sum of their deficit angles is more or equal to 2π

Boundary of the Gott Time Machine



Correlation functions on the boundary of Gott's TM

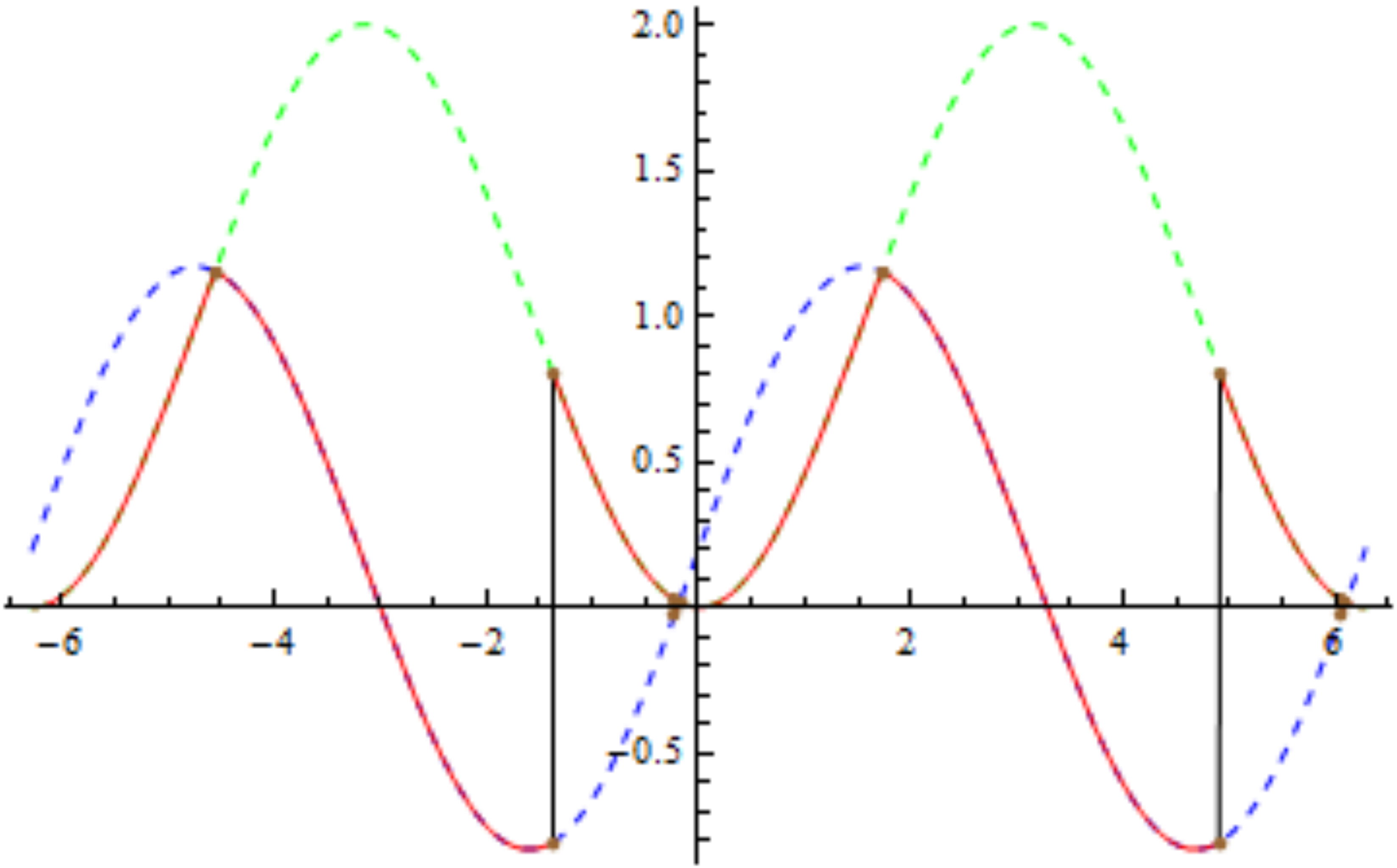
The inverse propagate is given by

$$F(\tau,\phi) = (\cos\tau - \cos\phi)\theta(-|\cos\tau - \cos\phi| + |\cos(\tau-\tau_0) - \cos(\phi-\phi_0)|)$$
$$+ (\cos(\tau-\tau_0) - \cos(\phi-\phi_0))\theta(|cos\tau - \cos\phi| - |\cos(\tau-\tau_0) - \cos(\phi-\phi_0)|)$$

$$\tau_0 = -2\arctan\left(\frac{\tanh\psi_0 \sin\frac{\alpha}{2}}{1+\tanh\psi_0\cos\frac{\alpha}{2}}\right)$$

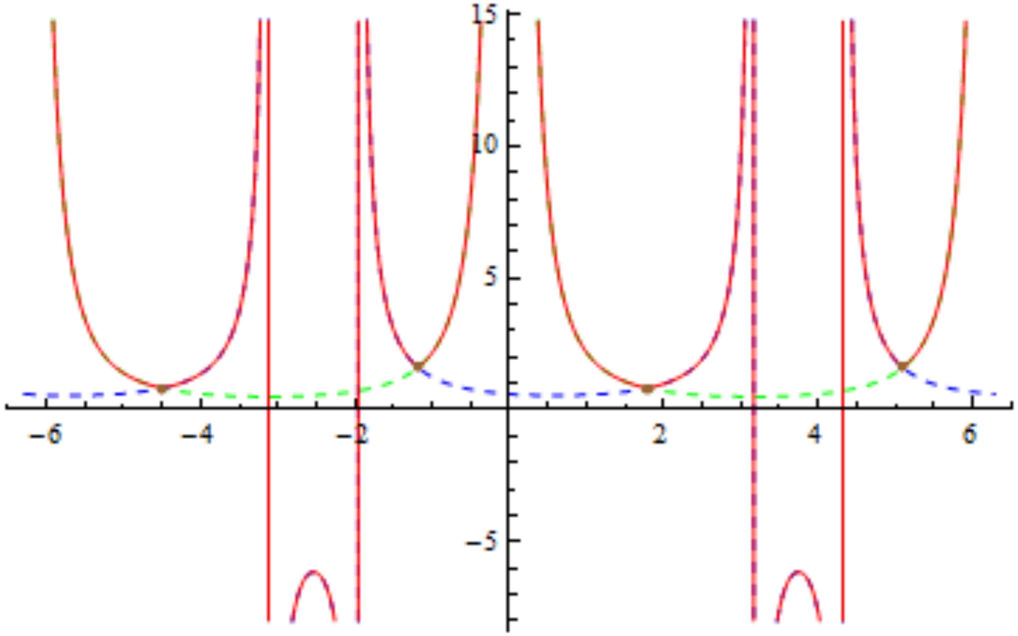
$$\phi_0 = -2\arctan\left(\frac{\sin\frac{\alpha}{2}}{\tanh\psi_0+\cos\frac{\alpha}{2}}\right)$$

Inverse correlation functions



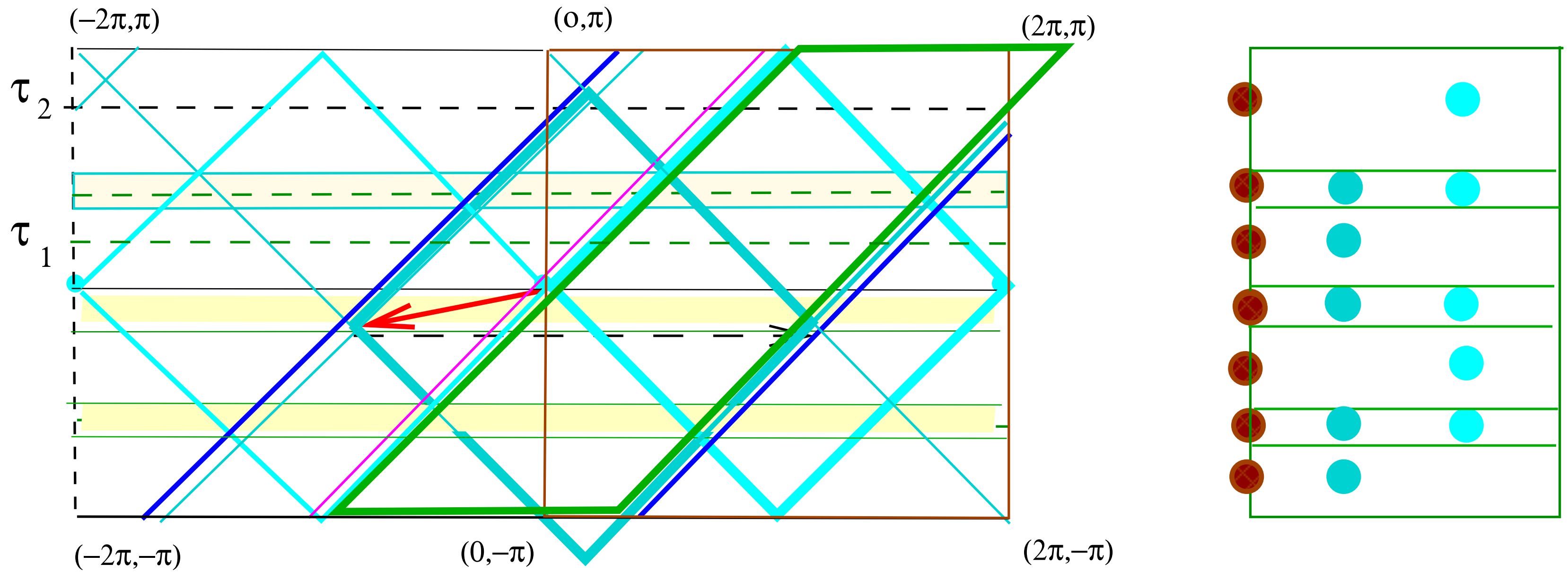
2-point Correlation Functions

$$G_\Delta^{(1)}(\tau,\phi) = \frac{\theta(-|\cos\tau - \cos\phi| + |\cos(\tau-\tau_0) - \cos(\phi-\phi_0)|)}{(\cos\tau - \cos\phi)^\Delta} + \frac{\theta(|cos\tau - \cos\phi| - |\cos(\tau-\tau_0) - \cos(\phi-\phi_0)|)}{(\cos(\tau-\tau_0) - \cos(\phi-\phi_0))^\Delta}$$



The causal picture of the boundary of the spacetime with 2 moving massive particle

The deficit angle is taken to be less then π

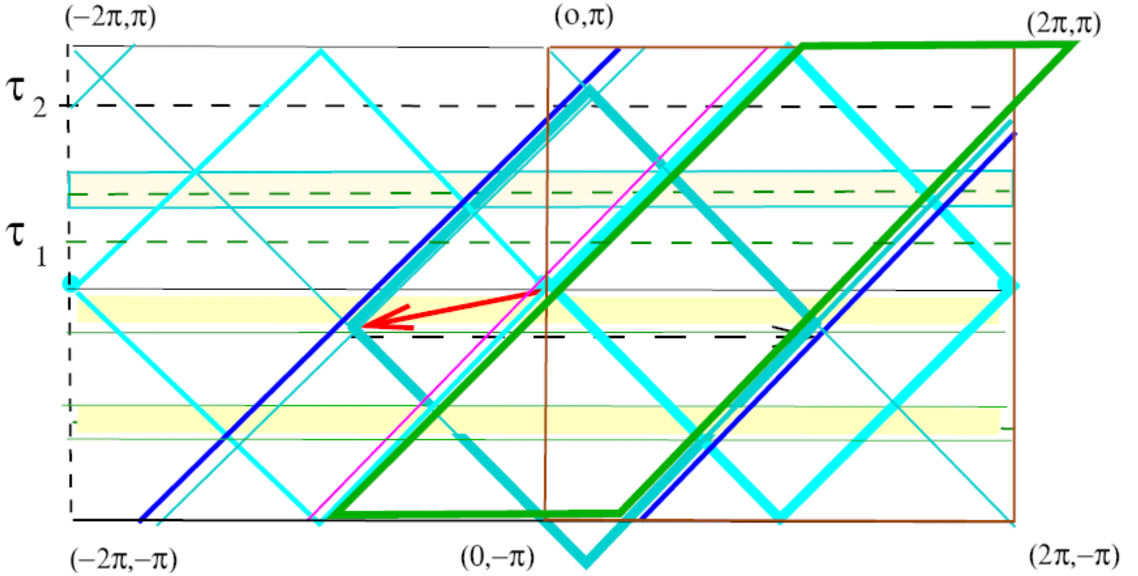


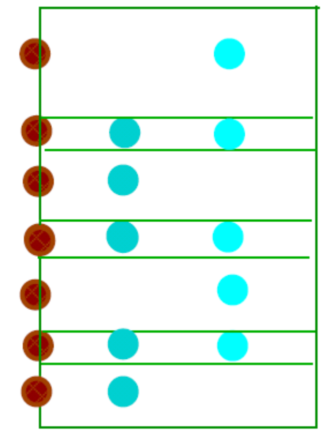
The area remaining after the cut-and-glue procedure (living space) is surrounded by the green parallelepiped

The light blue squares original poles of the Green function

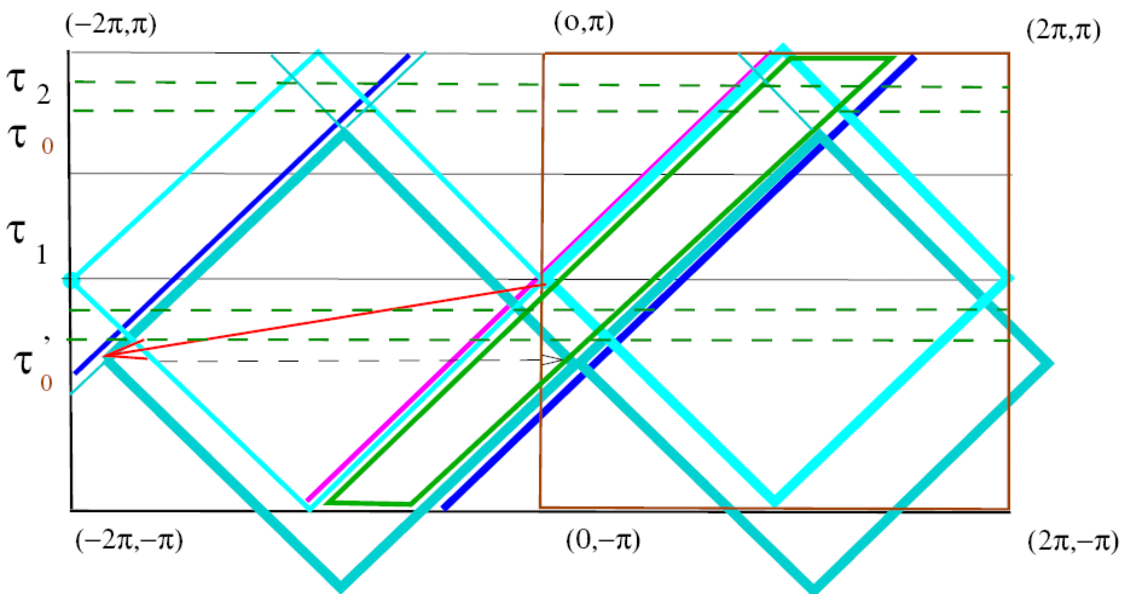
The marine squares additional poles of the Green function

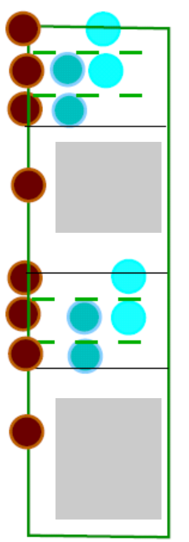
Number of zeros of the inverse propagator





No dark regions





Dark regions

Conclusion

Holographic Models of QGP formation

A. Shock waves collisions in AdS5

B. Infalling shell

C. 3-dim toy examples

Main achievements:

A. Multiplicity $$S_{data} \propto s_{NN}^{0.15}$$

B. Thermalization time

C. Conformal symmetry breaking produces infinite number of excitations