# Reflections on information paradox in Black Holes

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- General relativity has very interesting classical solutions - Black Holes. They are characterised by the total mass, electric charge and angular momentum.
- And it seems, they have no other hairs! Still, they can absorb lots of matter and information.
- (Note that, for simplicity, we will only discuss the Schwarzschild solution with zero charge and angular momentum.)
- It has been understood long time ago that an intrinsic entropy must be ascribed to the Black Holes if the laws of thermodynamics are to be satisfied.

- Bekenstein has found that everything works well with the entropy

$$S = \frac{\text{horizon area}}{4G} = 4\pi M^2 G$$

where $G$ is the gravitational constant, $M$ is the mass of the Black Hole, and its radius is $R = 2MG$.
- But what does it actually mean? What are the microstates?

- A bit later, Hawking proved that, in quantum theory, the Black Holes must emit radiation with temperature

$$T = \frac{1}{8\pi GM}$$

- Roughly, this is the effect of virtual pairs in presence of horizon.
- Compare with the Unruh effect for an accelerated observer!

Note that

combining $E = M$ and $S = 4\pi GM^2$

with the Hawking temperature

$T = \frac{1}{8\pi GM}$,

we see that the thermodynamic relation

$dE = TdS$

is perfectly satisfied!

- However, it also brought about a big problem. The Black Hole evaporates producing the thermal radiation, and the smaller it is, the faster it emits its energy. Finally, it must totally disappear.
- But there the information is gone? It seems to violate the unitarity of quantum evolution.
- There are different ways to proceed
  – abandon the unitarity – quite problematic but some people also try it in relation to the measurement problem, see works of Daniel Sudarsky, etc.
  – abandon equivalence principle – something special at the horizon (a brick wall, or whatever) – come back later
  – stable remnants (hugely, or even infinitely, degenerate states) and other exotic possibilities
  – the paradigm of black hole complementarity (Susskind, Thorlacius, Uglum, PRD48, 3743 (1993))

The Black Hole complementarity:

– The process of formation and evapuration of a Black Hole is unitary. The information is beeing released by the Hawking radiation.

– For a distant observer, the Black Hole is a system with discrete energy levels, and with the number of states corresponding to the Bekenstein entropy. His viewpoint can be described in terms of a stretched horizon which can absorb, thermalise and re-emit the information. Some of its physical properties can actually be understood from the general principles - see, for example, a book by Susskind and Lindesay.

– Outside the stretched horizon, the world can be described, to a good approximation, by already known physics.

– However, an in-falling observer feels nothing special at the horizon, in full accordance with the equivalence principle.

- Those two observers both have their own self-consistent descriptions of reality.
- The in-falling one cannot send a signal from inside the horizon to the distant colleague.
- And many thought experiments were discussed to show that, for example, nobody can see cloning of a quantum state which is prohibited by quantum mechanics.
- Interestingly, Black Hole evaporation must violate the baryon number conservation. For the distant observer, there should not be any surprise due to the Planck-scale temperatures at the stretched horizon. What about the in-falling one? There are GUT-scale quantum fluctuations in his frame! See, again the book by Susskind and Lindesay.

- This picture has recently been challenged, at least for old Black Holes.
- By "old" we mean that at least one half of the total entropy has been emitted. It was shown long time ago by Don Page that after this time – the Page time – practically all the information is released from the Black Hole. And therefore, the new, near-horizon radiation must be maximally entangled with the early Hawking radiation which is, to the moment, extremely far away from the parent Black Hole.

- In order to understand the relevant time and length scales, recall that $T \propto M^{-1}$, the emissive power $\propto T^4 \propto M^{-4}$ while the horizon area $\propto M^2$. It yields $\dot{M} \propto M^{-2}$, and therefore $t \propto M^3$.
- In other words,

$$t_{\mathrm{Page}} \approx \left( \frac{M}{m_{\mathrm{Planck}}} \right)^3 t_{\mathrm{Planck}}.$$

- For astrophysical Black Holes, this time is impractically huge. But this is a matter of fundamental principles!

- The essence of the problem is that the in-falling observer must see the featureless vacuum state while crossing the horizon of a large Black Hole. And the vacuum is a very regular thing with the maximal entanglement between the two – inside and outside horizon – parts.
- But, after the Page time, the outer near-horizon zone – i.e. the late Hawking radiation – must also be maximally entangled with the early radiation.
- We arrived at the violation of monogamy of entanglement!
- A possible way-out is to abandon the equivalence principle. Then, the lack of entanglement in the near-horizon zone translates to presence of energetic quanta, or the firewall. See Almheiri, Marolf, Polchinski, Sully, JHEP02(2013)062, or also Braunstein, arXiv:0907.1190v1 (energetic curtains), or may be even some earlier works.

- A possible solution was to simply identify the interior of the old Black Hole with its early radiation. This is uncomfortably non-local. And, as was argued by Rafael Bousso, to be useful, this map must adjust to any interactions, and therefore it should be impossible for the in-falling observer to excite the vacuum state. And this "frozen vacuum" violates the equivalence principle equally bad.
- There were also attempts (e.g. by Rafael Bousso) to resort to the strong complementarity by stating that this contradiction is not that much of a problem as long as the two observers cannot communicate their findings to each other.
- However, it did not work out perfectly because there still appears to be some time for the in-falling observer to change his mind and to turn back after making a very precise measurement of the Hawking radiation during the free fall.

- One could still try to save this proposal by stating that the contradictory measurements and inferences are
  – computationaly unfeasible (Harlow, Hayden)
  – extremely fine-grained (Susskind, and also see Papadodimas, Raju in terms of AdS/CFT)
  – akin to observing quantum superpositions of macroscopic worlds (Nomura, Varela, etal.)
- It would mean some new limitations for applicability of EFT description. However, then it would be nice to see some understandable reasons for such limitations. And also it looks uneasy to preserve the basic principles of quantum mechanics only by dividing the world into separate causal patches.

- One line of thought is that it is precisely our measurement of the quantum state of the early photons which produces the energetic quanta to kill the in-falling colleague. To make it more causal, the Einstein-Rosen bridges are invoked. This comes to the

  $$\mathrm{EPR} = \mathrm{ER}$$

  concept by Maldacena and Susskind.
- Of course, it touches upon the sensitive topics of the measurement problem...

- And one more proposal has been made by Stephen Hsu. He refers to an old observation due to Don Page that the back-reaction will produce a huge uncertainty in position and momentum of an evaporating Black Hole. And the idea is to use the many-worlds interpretation of quantum mechanics and to hope that, in the full picture, the process would manifestly be unitary.
- Of course, it again involves the interpretational issue, and moreover, impossibility to self-consistently choose one macroscopic branch might seem too radical even for many-worlds interpretation.

- Let us try to find a possible reason for the troubles without discussing interpretations of quantum mechanics.
- The Hawking quanta with typical wavelength of order

$$\lambda \approx \frac{M}{m_{\mathrm{Planck}}} l_{\mathrm{Planck}}$$

  travel to us through the quantum space-time foam.
- Let us assume that it induces the quantum uncertainty

$$\delta\lambda \approx l_{\mathrm{Planck}}.$$

- After travelling over $N$ wavelengths $L = N\lambda$ we have

$$\delta L \approx \sqrt{N} \cdot l_{\mathrm{Planck}}.$$

- It is easy to see that $\delta L$ reaches $\lambda$ when $L \approx t_{\mathrm{Page}}$.

- In other words, the problems appear when the information about the phase of the photon is totally lost.
- The purity of the Hawking radiation is lost due to interactions with space-time foam. Of course, it does not give us a full picture of interactions in the system. However, the paradox seems to be avoided due to the loss of coherence of the early radiation.
- The semi-classical description of quantum fields in a fixed background remains accurate enough in every region where it should be. However, attempting to self-consistently use it over huge distances might be the real problem. Tiny, almost evanescent errors, being made again and again, might eventually accumulate to sizable effects.

- Note that the Page time has naturally appeared above from a completely independent direction.
- Note also that sometimes it is argued that the real time scale of the problem is not the Page time but the scrambling time, i.e. the time which is required for thermalisation of the information by the Black Hole. Sekino and Susskind have estimated it as

$$t \approx \left( \frac{M}{m_{\mathrm{Planck}}} \log \frac{M}{m_{\mathrm{Planck}}} \right) t_{\mathrm{Planck}}.$$

- Can we reproduce this – scrambling – time scale from our analysis, too? Interestingly, the answer seems to be "yes".
- One can try to describe the effects of quantum gravitational foam in terms of a quantum dynamical semigroup for the density matrix

  $$\dot{\rho} = \hat{\mathcal{L}}\rho$$

  where the Lindblad operator contains the usual commutator with the Hamiltonian and also some additional terms inducing the decoherence which actually depend on the adopted level of coarse-graining.
- If our resolution scale is nearly Planckian, we estimate the matrix entries of the Lindblad operator of order $\mathfrak{l} \approx \frac{c}{\lambda} \approx \frac{m_{\rm Planck}}{M t_{\rm Planck}}$ .

- Assume that initially the photon was almost in a pure state with $p_1$ close to unity, and other states were present with quantum (gravitational) probabilities of order $p_j \approx \frac{m_{\mathrm{Planck}}}{M}$, or some mild power of it.
- And $\log p_j$ starts growing like $\propto \mathfrak{l} t$.
- We see that, after the scrambling time, we will have a perfect statistical mixture instead of the very pure state which we have started with.

- Let me repeat my conclusion.
  It very well might be so that, with the Black Hole information paradox, we do not need to abandon any of our basic physical principles. Instead, it might be necessary to understand that even a very precise model can give us totally wrong results being naively applied over huge enough time and distance scales.
- Whatever the final conclusion would be, it is very remarkable that the Black Holes again can teach us an interesting lesson about our fundamental theories.

Thank you for your attention!