Emergent Cosmology, Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

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Talk dedicated to the memory of Sergio del Campo, chilean cosmologist

Who passed away a month ago, and with whom I worked. He started theoretical cosmology in Chile.

Subject of this talk:

Here we will see how a unified picture of inflation and present dark energy can be consistent with a smooth, non singular origin of the universe, represented by the emergent scenario, presenting an attractive cosmological scenario. This is achieved by considering two non Riemannian measures or volume forms in the action. The motivation is:

– The early inflation, although solving many cosmological puzzles, like the horizon and flatness problems, cannot address the initial singularity problem;
– There is no explanation for the existence of two periods of exponential expansion with such wildly different scales – the inflationary phase and the present phase of slowly accelerated expansion of the universe.

The best known mechanism for generating a period of accelerated expansion is through the presence of some vacuum energy. In the context of a scalar field theory, vacuum energy density appears naturally when the scalar field acquires an effective potential $U_{\rm eff}$ which has flat regions so that the scalar field can "slowly roll"

and its kinetic energy can be neglected resulting in an energy-momentum

tensor $T_{\mu\nu} \simeq g_{\mu\nu} U_{\rm eff}$

we will study a unified scenario where both an inflation

and a slowly accelerated phase for the universe can appear naturally from the

existence of two flat regions in the effective scalar field potential which we derive systematically from a Lagrangian action principle. Namely, we start with a new kind of globally Weyl-scale invariant gravity-matter action within the first-order (Palatini) approach formulated in terms of two different non-Riemannian volume forms (integration measures)

$$S = \int d^4x\, \Phi_1(A) \Big[R + L^{(1)}\Big] + \int d^4x\, \Phi_2(B) \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big]$$

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, *i.e.*, generally covariant integration measure densities on the underlying space-time manifold:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; , \tag{2}$$

  defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields. $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$ in terms of the space-time metric $g_{\mu\nu}$.
- $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma^{\mu}_{\nu\lambda}$ is *a priori* independent of the metric $g_{\mu\nu}$. Note that in the second action term we have added a R^2 gravity term (again in the Palatini form). Let us recall that $R + R^2$ gravity within the second order formalism (which was also the first inflationary model)

- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field

Alternative realization of a non Riemannian measure, from a mapping of two spaces:

density can be built out of four auxiliary scalar fields φ^i $(i = 1, 2, 3, 4)$:

$$\Phi(\varphi) = \frac{1}{4!}\varepsilon^{\mu\nu\kappa\lambda}\varepsilon_{ijkl}\partial_\mu\varphi^i\partial_\nu\varphi^j\partial_\kappa\varphi^k\partial_\lambda\varphi^l \, .$$

$\Phi(\varphi)$ is a scalar density under general coordinate transformations.

Ideas from where can we get 4 scalars, for example from Cederwall's

Doubling of space time, by adding the twiddle coordinates which are scalars w/r to the "normal space" and then define a "brane " where the twiddle coordinates are a functions of un-twiddle coordinates and Jacobian from the mapping defines measure of integration?,

define X^M to denote coordinates and dual coordinates

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} .$$

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \quad , \quad V(\varphi) = f_1 \exp\{-\alpha\varphi\} \; , \tag{3}$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) \quad , \quad U(\varphi) = f_2 \exp\{-2\alpha\varphi\} \; , \tag{4}$$

where α, f_1, f_2 are dimensionful positive parameter, whereas b is a dimensionless one.

– $\Phi(H)$ indicates the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu H_{\nu\kappa\lambda} \; , \tag{5}$$

whose presence is crucial for non-triviality of the model.

The scalar potentials have been chosen in such a way that the original action (1) is invariant under global Weyl-scale transformations:

$$\begin{aligned} g_{\mu\nu} \to \lambda g_{\mu\nu} \;\; , \;\; \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda \;\; , \\ A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \;\; , \;\; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \;\; , \;\; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; . \end{aligned} \tag{6}$$

The equations of motion resulting from the action (1) are as follows. Variation of (1) w.r.t. affine connection $\Gamma^\mu_{\nu\lambda}$:

$$\int d^4 x \sqrt{-g} g^{\mu\nu} \Big(\frac{\Phi_1}{\sqrt{-g}} + 2\epsilon \frac{\Phi_2}{\sqrt{-g}} R \Big) \left(\nabla_\kappa \delta \Gamma^\kappa_{\mu\nu} - \nabla_\mu \delta \Gamma^\kappa_{\kappa\nu} \right) = 0 \tag{7}$$

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}\right) , \tag{8}$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R) g_{\mu\nu} \quad , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \quad , \quad \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}} . \tag{9}$$

Variation of the action (1) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Big[R + L^{(1)}\Big] = 0 \quad , \quad \partial_\mu \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0 \quad , \quad \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0 , \tag{10}$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} \;\; , \;\; R + L^{(1)} = -M_1 = \text{const} \;\; , \;\; L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} \tag{11}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimenionless integration constants. The appearance of M_1, M_2 signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance under (6) due to the scale non-invariant solutions (second and third ones) in (11).

Varying (1) w.r.t. $g_{\mu\nu}$ and using relations (11) we have:

$$\chi_1 \Big[R_{\mu\nu} + \frac{1}{2}\Big(g_{\mu\nu} L^{(1)} - T^{(1)}_{\mu\nu}\Big)\Big] - \frac{1}{2}\chi_2 \Big[T^{(2)}_{\mu\nu} + g_{\mu\nu}\Big(\epsilon R^2 + M_2\Big) - 2R\, R_{\mu\nu}\Big] = 0 , \tag{12}$$

where χ_1 and χ_2 are defined in (9), and $T^{(1,2)}_{\mu\nu}$ are the energy-momentum tensors of the scalar field Lagrangians with the standard definitions:

of the scalar field Lagrangians with the standard definitions:

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2\frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} \; . \tag{13}$$

Taking the trace of Eqs.(12) and using again second relation (11) we solve for the scale factor χ_1:

$$\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1} \; , \tag{14}$$

where $T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}$.

Using second relation (11) Eqs.(12) can be put in the Einstein-like form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} g_{\mu\nu} \left(L^{(1)} + M_1\right) + \frac{1}{2\Omega}\left(T^{(1)}_{\mu\nu} - g_{\mu\nu} L^{(1)}\right)$$
$$+ \frac{\chi_2}{2\chi_1 \Omega}\left[T^{(2)}_{\mu\nu} + g_{\mu\nu}\left(M_2 + \epsilon (L^{(1)} + M_1)^2\right)\right] , \tag{15}$$

where:

$$\Omega = 1 - \frac{\chi_2}{\chi_1} 2\epsilon \left(L^{(1)} + M_1\right) \; . \tag{16}$$

Let us note that (9), upon taking into account second relation (11) and (16), can be written as:

$$\bar{g}_{\mu\nu} = \chi_1 \Omega \, g_{\mu\nu} \; . \tag{17}$$

Now, we can bring Eqs.(15) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (17), *i.e.*, the Einstein-frame equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu} \qquad (18)$$

with energy-momentum tensor corresponding (according to (13)) to the following effective (Einstein-frame) scalar field Lagrangian:

$$L_{\rm eff} = \frac{1}{\chi_1 \Omega}\Big\{L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \Omega}\Big[L^{(2)} + M_1 + \epsilon(L^{(1)} + M_1)^2\Big]\Big\} . \qquad (19)$$

In order to explicitly write $L_{\rm eff}$ in terms of the Einstein-frame metric $\bar{g}_{\mu\nu}$ (17) we use the short-hand notation for the scalar kinetic term:

$$X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \qquad (20)$$

and represent $L^{(1,2)}$ in the form:

$$L^{(1)} = \chi_1 \Omega X - V \quad , \quad L^{(2)} = \chi_1 \Omega\, b e^{-\alpha\varphi} X + U , \qquad (21)$$

with V and U as in (3)-(4).

From Eqs.(14) and (16), taking into account (21), we find:

$$\frac{1}{\chi_1 \Omega} = \frac{(V - M_1)}{2\chi_2\Big[U + M_2 + \epsilon(V - M_1)^2\Big]}\Big[1 - \chi_2\Big(\frac{b e^{-\alpha\varphi}}{V - M_1} - 2\epsilon\Big)X\Big] . \qquad (22)$$

Upon substituting expression (22) into (19) we arrive at the explicit form for the Einstein-frame scalar Lagrangian:

$$L_{\rm eff} = A(\varphi)X + B(\varphi)X^2 - U_{\rm eff}(\varphi) , \qquad (23)$$

where:

$$A(\varphi) \equiv 1 + \left[\frac{1}{2} b e^{-\alpha\varphi} - \epsilon (V - M_1)\right] \frac{V - M_1}{U + M_2 + \epsilon (V - M_1)^2}$$

$$= 1 + \left[\frac{1}{2} b e^{-\alpha\varphi} - \epsilon \left(f_1 e^{-\alpha\varphi} - M_1\right)\right] \frac{f_1 e^{-\alpha\varphi} - M_1}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon (f_1 e^{-\alpha\varphi} - M_1)^2} , \quad (24)$$

and

$$B(\varphi) \equiv \chi_2 \frac{\epsilon \Big[U + M_2 + (V - M_1) b e^{-\alpha\varphi}\Big] - \frac{1}{4} b^2 e^{-2\alpha\varphi}}{U + M_2 + \epsilon (V - M_1)^2}$$

$$= \chi_2 \frac{\epsilon \Big[f_2 e^{-2\alpha\varphi} + M_2 + (f_1 e^{-\alpha\varphi} - M_1) b e^{-\alpha\varphi}\Big] - \frac{1}{4} b^2 e^{-2\alpha\varphi}}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon (f_1 e^{-\alpha\varphi} - M_1)^2} , \quad (25)$$

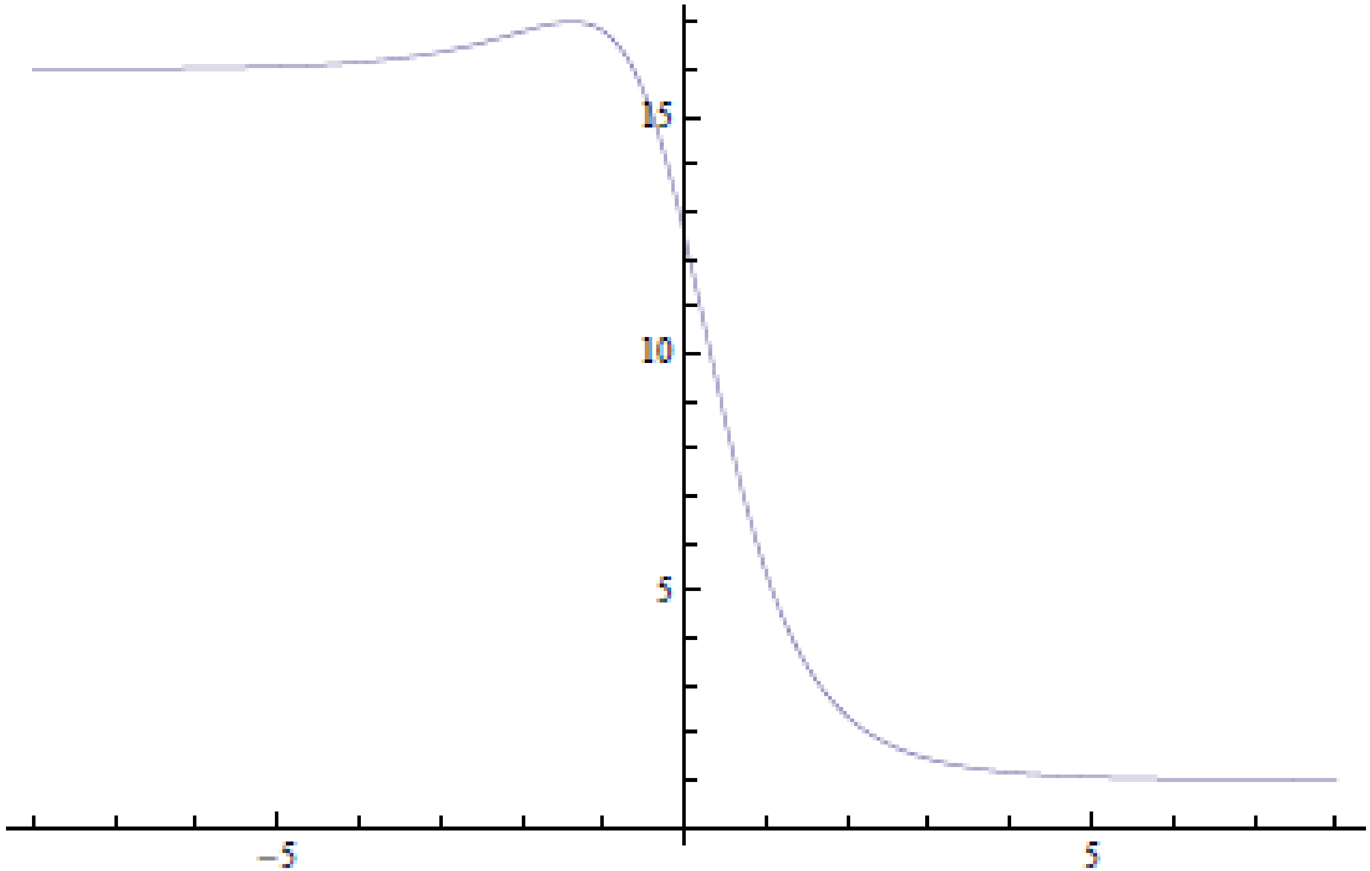
whereas the effective scalar field potential reads:

$$U_{\rm eff}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2 \Big[U + M_2 + \epsilon (V - M_1)^2\Big]} = \frac{\left(f_1 e^{-\alpha\varphi} - M_1\right)^2}{4\chi_2 \left[f_2 e^{-2\alpha\varphi} + M_2 + \epsilon (f_1 e^{-\alpha\varphi} - M_1)^2\right]} , \quad (26)$$

where the explicit form of V and U (3)-(4) are inserted.

Let us recall that the dimensionless integration constant χ_2 is the ratio of the original second non-Riemannian integration measure to the standard Riemannian one (9).

Shape of the effective scalar potential $U_{\text{eff}}(\varphi)$ (26) for $M_1 < 0$.



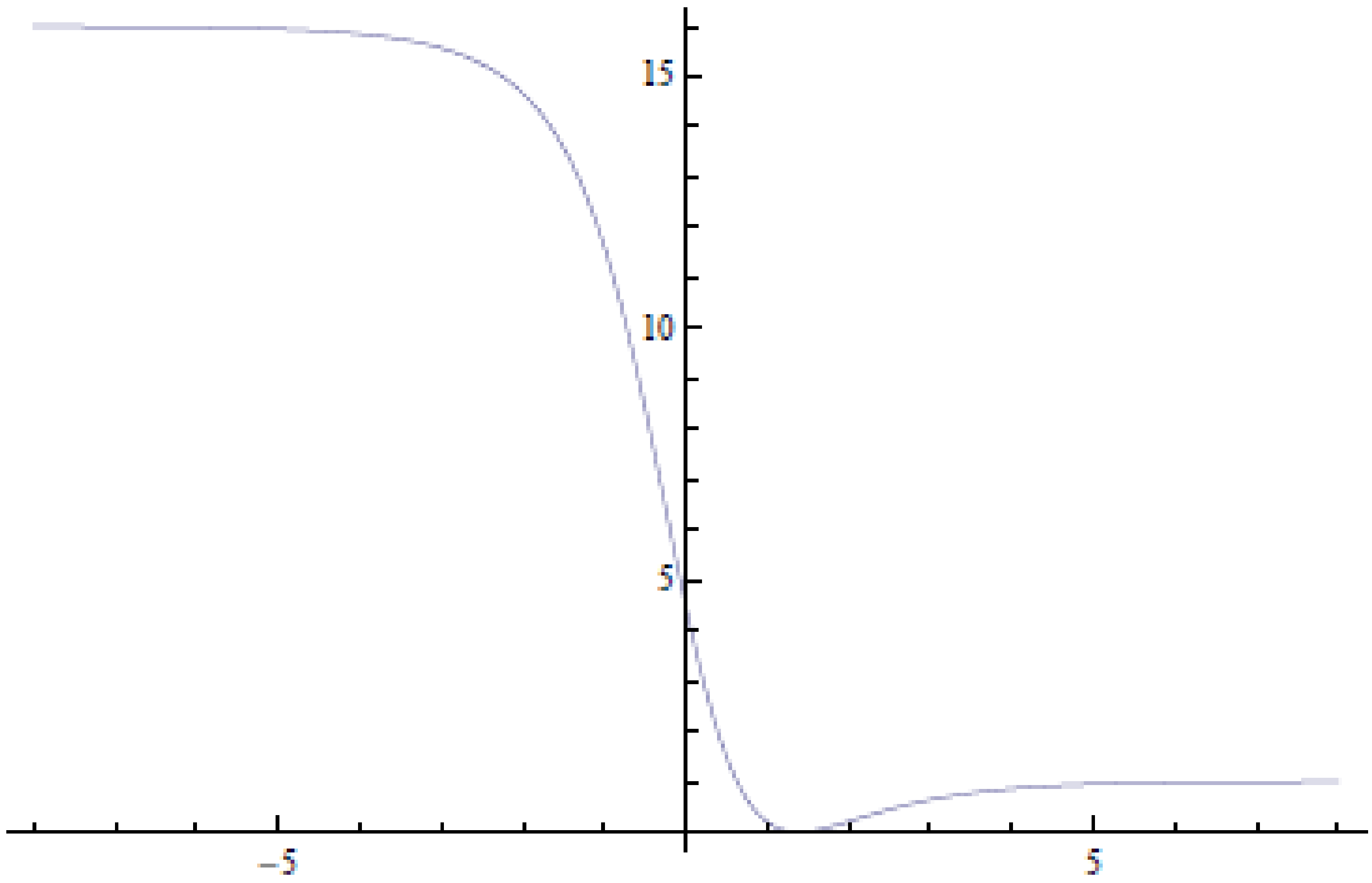
Interesting feature of the effective potential in the case no R^2 terms

are introduced, which is that the bump showed in the picture raises above the first flat region (relevant for the early universe) as much as the

second flat region (relevant for the present universe) is above zero! . There is a hint of the present universe which appears in the early universe.

For the other sign, we get a shape similar to that of the Starobinsky model, which provides a good description of inflation, but now, the new thing is the additional flat region that can take care of the present dark energy!, see:

Shape of the effective scalar potential $U_{\text{eff}}(\varphi)$ (26) for $M_1 > 0$.



3 Flat Regions of the Effective Scalar Potential

Depending on the sign of the integration constant M_1 we obtain two types of shapes for the effective scalar potentail $U_{\rm eff}(\varphi)$ (26) depicted on Fig.1. and Fig.2.

The crucial feature of $U_{\rm eff}(\varphi)$ is the presence of two very large flat regions – for negative and positive values of the scalar field φ. For large negative values of φ we have for the effective potential and the coefficient functions in the Einstein-frame scalar Lagrangian (23)-(26):

$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1+\epsilon f_1^2/f_2)} \; , \tag{27}$$

$$A(\varphi) \simeq A_{(-)} \equiv \frac{1+\frac{1}{2}bf_1/f_2}{1+\epsilon f_1^2/f_2} \;\; , \;\; B(\varphi) \simeq B_{(-)} \equiv -\chi_2 \frac{b^2/4f_2 - \epsilon(1+bf_1/f_2)}{1+\epsilon f_1^2/f_2} \; . \tag{28}$$

In the second flat region for large positive φ:

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)} \; , \tag{29}$$

$$A(\varphi) \simeq A_{(+)} \equiv \frac{M_2}{M_2+\epsilon M_1^2} \quad , \quad B(\varphi) \simeq B_{(+)} \equiv \epsilon\chi_2 \frac{M_2}{M_2+\epsilon M_1^2} \; . \tag{30}$$

From the expression for $U_{\rm eff}(\varphi)$ (26) and the figures 1 and 2 we see that now we have an explicit realization of quintessential inflation scenario. The flat regions (27)-(28) and (29)-(30) correspond to the evolution of the early and the late universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{f_1^2/f_2}{1+\epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1+\epsilon M_1^2/M_2}\,, \tag{31}$$

which makes the vacuum energy density of the early universe $U_{(-)}$ much bigger than that of the late universe $U_{(+)}$ (cf. (27), (29)). The inequality (31) is equivalent to the requirements:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2} \quad , \quad |\epsilon|\frac{M_1^2}{M_2} \ll 1\,. \tag{32}$$

In particular, if we choose the scales of the scale symmetry breaking integration constants $M_1 \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where M_{EW}, M_{Pl} are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density $U_{(+)} \sim M_1^2/M_2$ of the order $M_{EW}^8/M_{Pl}^4 \sim 10^{-120}M_{Pl}^4$, which is the right order of magnitude for the present epoche's vacuum energy density as already recognized in Ref.[27]. On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2}M_{Pl})^4$, then together with the above choice of order of magnitudes for $M_{1,2}$ the inequalities (32) will be satisfied as well and the order of magnitude of the vacuum energy density of the early universe $U_{(-)}$ (27) becomes $U_{(-)} \sim f_1^2/f_2 \sim 10^{-8}M_{Pl}^4$ which conforms to the BICEP2 data [5] implying the energy scale of inflation of order $10^{-2}M_{Pl}$.

Before proceeding to the derivation of the non-singular "emergent universe" solution describing an initial phase of the universe evolution preceeding the inflationary phase, let us briefly sketch how the present non-Riemannian-measure-modified gravity-matter theory meets the conditions for the validity of the "slow-roll" approximation [6] when φ evolves on the flat region of the effective potential corresponding to the early universe (27)-(28).

To this end let us recall the standard Friedman-Lemaitre-Robertson-Walker space-time metric [26]:

$$ds^2 = -dt^2 + a^2(t)\Big[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big] \tag{33}$$

and the associated Friedman equations (recall the presently used units $G_{\rm Newton} = 1/16\pi$):

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \, , \tag{34}$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{3}{4}B(\varphi)\,\dot{\varphi}^4 + U_{\rm eff}(\varphi) \, , \tag{35}$$

$$p = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{1}{4}B(\varphi)\,\dot{\varphi}^4 - U_{\rm eff}(\varphi) \tag{36}$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$. Henceforth the dots indicate derivatives with respect to the time t.

Let us now consider the standard "slow-roll" parameters [7]:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \quad , \quad \eta \equiv -\frac{\ddot{\varphi}}{H\,\dot{\varphi}} \, , \tag{37}$$

where ε measures the ratio of the scalar field kinetic energy relative to its total energy density and η measures the ratio of the fields acceleration relative to the "friction" ($\sim 3H\,\dot{\varphi}$) term in the pertinent scalar field equations of motion:

$$\ddot{\varphi}\,(A + 3B\,\dot{\varphi}^2) + 3H\,\dot{\varphi}\,(A + B\,\dot{\varphi}^2) + U'_{\rm eff} + \frac{1}{2}A'\,\dot{\varphi}^2 + \frac{3}{4}B'\,\dot{\varphi}^4 = 0 \, , \tag{38}$$

with primes indicating derivatives w.r.t. φ.

In the slow-roll approximation one ignores the terms with $\ddot{\varphi}$, $\dot{\varphi}^2$, $\dot{\varphi}^3$, $\dot{\varphi}^4$ so that the φ-equation of motion (38) and the second Friedman Eq.(34) reduce to:

$$3AH\,\dot{\varphi} + U'_{\rm eff} = 0 \quad , \quad H^2 = \frac{1}{6}U_{\rm eff} \, . \tag{39}$$

Now, using the fact that φ evolves on a flat region of $U_{\rm eff}$ we deduce that $H \equiv \dot{a}/a \simeq$ const, so that $a(t)$ grows exponentially with time and, thus, in the second Eq.(39) the spatial curvature term K/a^2 is ignored. Consistency of the slow-roll approximation implies for the slow-roll parameters (37), taking into account (39), the following inequalities:

$$\varepsilon \simeq \frac{1}{A}\left(\frac{U'_{\rm eff}}{U_{\rm eff}}\right)^2 \ll 1 \quad , \quad \eta \simeq \frac{2}{A}\,\frac{U''_{\rm eff}}{U_{\rm eff}} - \varepsilon - \frac{2A'}{A^{3/2}}\sqrt{\varepsilon} \;\to\; \frac{2}{A}\,\frac{U''_{\rm eff}}{U_{\rm eff}} \ll 1 \, . \tag{40}$$

Since now φ evolves on the flat region of $U_{\rm eff}$ for large negative values (27), the Lagrangian coefficient function $A(\varphi) \simeq A_{(-)}$ as in (28) and the gradient of the effective scalar potential is:

$$U'_{\rm eff} \simeq -\frac{\alpha f_1 M_1 e^{\alpha\varphi}}{2\chi_2 f_2 (1+\epsilon f_1^2/f_2)^2} \; , \tag{41}$$

which yields for the slow-roll parameter ε (40):

$$\varepsilon \simeq \frac{4\alpha^2 M_1^2 e^{2\alpha\varphi}}{f_1^2 (1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)} \ll 1 \quad \text{for large negative } \varphi \; . \tag{42}$$

Similarly, for the second slow-roll parameter we have:

$$|\frac{2}{A}\,\frac{U''_{\rm eff}}{U_{\rm eff}}| \simeq \frac{4\alpha^2 M_1 e^{\alpha\varphi}}{f_1 (1+bf_1/2f_2)} \ll 1 \quad \text{for large negative } \varphi \; . \tag{43}$$

The value of φ at the end of the slow-roll regime $\varphi_{\rm end}$ is determined from the condition $\varepsilon \simeq 1$ which through (42) yields:

$$e^{-2\alpha\varphi_{\rm end}} \simeq \frac{4\alpha^2 M_1^2}{f_1^2(1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)} \tag{44}$$

The amount of inflation when φ evolves from some initial value $\varphi_{\rm in}$ to the end-point of slow-roll inflation $\varphi_{\rm end}$ is determined through the expression for the *e-foldings* N []:

$$N = \int_{\varphi_{\rm in}}^{\varphi_{\rm end}} H\,dt = \int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{H}{\dot\varphi} d\varphi \simeq -\int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{3H^2 A}{U'_{\rm eff}} d\varphi \simeq -\int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{AU_{\rm eff}}{2U'_{\rm eff}} d\varphi\,, \tag{45}$$

where Eqs.(39) are used. Substituting (27), (28) and (41) into (45) yields an expression for N which together with (44) allows for the determination of $\varphi_{\rm in}$:

$$N \simeq \frac{f_1(1+bf_1/f_2)}{4\alpha^2 M_1}\left(e^{-\alpha\varphi_{\rm in}} - e^{-\alpha\varphi_{\rm end}}\right)\,. \tag{46}$$

4 Non-Singular Emergent Universe Solution

We will now show that under appropriate restrictions on the parameters there exist an epoch preceeding the inflationary phase. Namely, we derive an explicit cosmological solution of the Einstein-frame system with effective scalar field Lagrangian (23)-(26) describing a non-singular "emergent universe" [15] when the scalar field evolves on the first flat region for large negative φ (27). For previous studies of "emergent universe" scenarios within the context of the less general modified-measure gravity-matter theories with one non-Riemannian and one standard Riemannian integration measures, see Ref.[21].

Emergent universe is defined through the standard Friedman-Lemaitre-Robertso Walker space-time metric (33) as a solution of (34) subject to the condition on the Hubble parameter H:

$$H=0 \quad \to \quad a(t)=a_0=\text{const}\,,\;\; \rho+3p=0 \quad , \quad \frac{K}{a_0^2}=\frac{1}{6}\rho\;(=\text{const})\,, \qquad (47)$$

with ρ and p as in (35)-(36):

The emergent universe condition (47) implies that the φ-velocity $\dot\varphi \equiv \dot\varphi_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2}B_{(-)}\,\dot\varphi_0^4 + 2A_{(-)}\,\dot\varphi_0^2 - 2U_{(-)} = 0 \qquad (48)$$

(with notations as in (27)-(28)), whose solution read:

$$\dot\varphi_0^2 = -\frac{2}{3B_{(-)}}\Big[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\Big]\,. \qquad (49)$$

To analyze stability of the present emergent universe solution:

$$a_0^2 = \frac{6k}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2} A_{(-)} \dot{\varphi}_0^2 + \frac{3}{4} B_{(-)} \dot{\varphi}_0^4 + U_{(-)} \, , \tag{50}$$

with $\dot{\varphi}_0^2$ as in (49), we perturb Friedman Eqs.(34) and the expressions for ρ, p (35)-(36) w.r.t. $a(t) = a_0 + \delta a(t)$ and $\dot{\varphi}(t) = \dot{\varphi}_0 + \delta\dot{\varphi}(t)$, but keep the effective potential on the flat region $U_{\rm eff} = U_{(-)}$:

$$\frac{\delta \ddot{a}}{a_0} + \frac{1}{12}(\delta\rho + 3\delta p) \quad , \quad \delta\rho = -\frac{2\rho_0}{a_0}\delta a \tag{51}$$

$$\delta\rho = \left(A_{(-)} \dot{\varphi}_0 + 3B_{(-)} \dot{\varphi}_0^3 \right) \delta\dot{\varphi} = -\frac{2\rho_0}{a_0}\delta a \quad , \quad \delta p = \left(A_{(-)} \dot{\varphi}_0 + B_{(-)} \dot{\varphi}_0^3 \right) \delta\dot{\varphi} \tag{52}$$

From the first Eq.(52) expressing $\delta\dot{\varphi}$ as function of δa and substituting into the first Eq.(51) we get a harmonic oscillator type equation for δa:

$$\delta \ddot{a} + \omega^2 \delta a = 0 \quad , \quad \omega^2 \equiv \frac{2}{3}\rho_0 \frac{\pm\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}}{A \mp 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}} \, , \tag{53}$$

where:

$$\rho_0 \equiv \frac{1}{2}\dot{\varphi}_0^2 \left[A_{(-)} + 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}} \right] , \tag{54}$$

with $\dot{\varphi}_0^2$ from (49). Thus, for existence and stability of the emergent universe solution we have to choose the upper signs in (49), (53) and we need the conditions:

$$A_{(-)}^2 + 3B_{(-)}U_{(-)} > 0 \quad , \quad A_{(-)} - 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}]} > 0 \,. \tag{55}$$

The latter yield the following constraint on the coupling parameters:

$$\max\Big\{-2\,,\, -8(1+3\epsilon f_1^2/f_2)\Big[1 - \sqrt{1 - \frac{1}{4(1+3\epsilon f_1^2/f_2)}}\Big]\Big\} < b\frac{f_1}{f_2} < -1\,, \tag{56}$$

in particular, implying that $b < 0$. The latter means that both terms in the original matter Lagrangian $L^{(2)}$ (4) appearing multiplied by the second non-Riemannian integration measure density Φ_2 (2) must be taken with "wrong" signs in order to have a consistent physical Einstein-frame theory (23)-(25) possessing a non-singular emergent universe solution.

For $\epsilon > 0$, since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the first flat region of the effective scalar potential, *i.e.*, the vacuum energy density in the early universe, must be large (cf. (31)), we find that the lower end of the interval in (56) is very close to the upper end, *i.e.*, $b\frac{f_1}{f_2} \simeq -1$.

In the present paper we have constructed a new kind of gravity-matter theory defined in terms of two different non-Riemannian volume-forms (generally covariant integration measure densities) on the space-time manifold, where the Einstein-Hilbert term R, its square R^2, the kinetic and the potential terms in the pertinent cosmological scalar field (a "dilaton") couple to each of the non-Riemannian integration measures in a manifestly globally Weyl-scale invariant form. The principal results are as follows:

- Dynamical spontaneous symmetry breaking of the global Weyl-scale invariance.
- In the physical Einstein frame we obtain an effective scalar field potential with *two flat regions* – one corresponding to the early universe evolution and a second one for the present slowly acelerating phase of the universe.
- The flat region of the effective scalar potential appropriate for describing the early universe allows for the existence of a *non-singular "emergent"* type beginning of the universe' evolution. This "emergent" phase is followed by the inflationary phase, which in turn is followed by a period, where the scalar field drops from its high energy density state to the present slowly accelerating phase of the universe.

The flatness of the effective scalar potential in the high energy density region makes the slow rolling inflation regime possible.

The presence of the emergent universe' phase preceeding the inflationary phase has observable consequences for the low CMB multipoles as has been recently shown in Ref.[29]. Therefore, a full analysis of the CMB results in the context of the present model should involve not only the classical "slow-roll" formalism, but also the "super-inflation" one, which describes the transition from the emergent universe to the inflationary phase.

Furthermore, it would be nice if we could apply the dynamical systems

analysis explained by Professor Marek Sydlowski concerning dynamical system to a rigorous analysis of all the stages of the cosmological scenario developed here: emergent universe, transition from emergent universe to inflation (period of super inflation), slow roll regime, etc.

For references, 1. look at this paper we just wrote

- arXiv: 1408.5344 astro.ph.CO (which appeared only monday this week), with my colleagues Alexander Kaganovich, Emil Nissimov and Svetlana Pacheva, entitled

Emergent Cosmology, Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

2. also look at a previous paper and references in both papers

Unification of Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

Eduardo Guendelman, Emil Nissimov, Svetlana Pacheva, Jul 23, 2014

e-Print: arXiv:1407.6281 [hep-th]

Applications of alternative measures, next talk is about with point (iii)

- (i) Study of $D = 4$-dimensional models of gravity and matter fields containing the new measure of integration (1), which appears to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, *etc.*
- (ii) Study of a new type of string and brane models based on employing of a modified world-sheet/world-volume integration measure. It allows for the appearance of new types of objects and effects like, for example, a spontaneously induced variable string tension.
- (iii) Studying modified supergravity models. Here we will find some outstanding new features: (a) the cosmological constant arises as an arbitrary integration constant, totally unrelated to the original parameters of the action, and (b) spontaneously breaking of local supersymmetry invariance.

The Supergravity modified measure

Action has the general form like the second piece of the action , with the second measure and the H field……….wait for Emil’s talk