

Regge Quantum Gravity

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- ▶ Path integral quantization of GR with matter based on Regge calculus such that a triangulation $T(M)$ of the spacetime manifold M is taken as the fundamental structure.
- ▶ If N is the number of cells of $T(M)$, then for $N \gg 1$, $T(M)$ looks like the smooth manifold M .
- ▶ It is not necessary to define the smooth limit $N \rightarrow \infty$. Instead, we need the large- N asymptotics of the observables.
- ▶ Semiclassical limit will be described by the effective action $\Gamma(L)$, which is computed by using the effective action equation from QFT, in the limit $L_\epsilon \gg l_P$.
- ▶ Inspired by the *spin-cube* models, which are generalizations of the *spin-foam* models.

Regge state-sum models

- ▶ The fundamental DOF are the edge-lengths $L_\epsilon \geq 0$, and

$$Z = \int_{D_E} \prod_{\epsilon=1}^E dL_\epsilon \mu(L) \exp(iS_{RC}(L)/l_P^2),$$

where

$$S_{RC} = - \sum_{\Delta=1}^F A_\Delta(L) \theta_\Delta(L) + \Lambda_c V_4(L),$$

is the Regge action with a cosmological constant. $D_E \subset \mathbf{R}_+^E$ such that the triangle inequalities hold.

- ▶ We choose the following PI measure

$$\mu(L) = e^{-V_4(L)/L_0^4},$$

where L_0 is a free parameter.

- ▶ We also introduce a classical CC length scale L_c such that

$$\Lambda_c = \pm 1/L_c^2.$$

Regge state-sum models

- ▶ We will use the *effective action* in order to determine the quantum corrections.
- ▶ EA is different from the *Wilsonian* approach to quantization which is used in QRC and CDT.
- ▶ In WQ approach

$$Z(g, \lambda) = \int_{D_E} \prod_{\epsilon=1}^E dL_\epsilon \mu(L) \exp [igS_R(L)/l_0^2 + i\lambda V_4(L)/l_0^4] ,$$

and one looks for points (g^*, λ^*) where Z'' diverges.

- ▶ In the vicinity of a critical point the correlation length diverges \Leftrightarrow transition from the discrete to a continuum theory.
- ▶ The semiclassical limit in WQ corresponds to the strong-coupling region $|g| \geq 1 \Rightarrow$ can be studied only numerically.

Effective action equation

- ▶ Let $\phi : M \rightarrow \mathbf{R}$ and $S(\phi) = \int_M d^4x \mathcal{L}(\phi, \partial\phi)$ a QFT flat-spacetime action. The effective action $\Gamma(\phi)$ is determined by the integro-differential equation

$$e^{i\Gamma(\phi)/\hbar} = \int \mathcal{D}h \exp \left[\frac{i}{\hbar} S(\phi + h) - \frac{i}{\hbar} \int_M d^4x \frac{\delta\Gamma}{\delta\phi(x)} h(x) \right].$$

- ▶ A solution $\Gamma(\phi) \in \mathbf{C}$, so that a Wick rotation is used to obtain $\Gamma(\phi) \in \mathbf{R}$: solve the Euclidean equation

$$e^{-\Gamma_E(\phi)/\hbar} = \int \mathcal{D}h \exp \left[-\frac{1}{\hbar} S_E(\phi + h) + \frac{1}{\hbar} \int_M d^4x \frac{\delta\Gamma_E}{\delta\phi(x)} h(x) \right],$$

and then put $x_0 = -it$ in $\Gamma_E(\phi)$.

- ▶ Wick rotation is equivalent to $\Gamma(\phi) \rightarrow \text{Re}\Gamma(\phi) + \text{Im}\Gamma(\phi)$

- ▶ In the case of a Regge state-sum model

$$e^{i\Gamma(L)/l_P^2} = \int_{D_E(L)} d^E x \mu(L+x) e^{iS_{Rc}(L+x)/l_P^2 - i \sum_{\epsilon=1}^E \Gamma'_\epsilon(L)x_\epsilon/l_P^2},$$

where $l_P^2 = G_N \hbar$ and $D_E(L)$ is a subset of \mathbf{R}^E obtained by translating D_E by a vector $-L$.

- ▶ $D_E(L) \subseteq [-L_1, \infty) \times \cdots \times [-L_E, \infty)$.
- ▶ Semiclassical solution

$$\Gamma(L) = S_{Rc}(L) + l_P^2 \Gamma_1(L) + l_P^4 \Gamma_2(L) + \cdots,$$

where $L_\epsilon \gg l_P$ and

$$|\Gamma_n(L)| \gg l_P^2 |\Gamma_{n+1}(L)|.$$

Perturbative solution

- ▶ Let $L_\epsilon \rightarrow \infty$, then $D_E(L) \rightarrow \mathbf{R}^E$ and

$$e^{i\Gamma(L)/l_P^2} \approx \int_{\mathbf{R}^E} d^E x \mu(L+x) e^{iS_{RC}(L+x)/l_P^2 - i \sum_{\epsilon=1}^E \Gamma'_\epsilon(L)x_\epsilon/l_P^2}.$$

- ▶ The reason is $D_E(L) \approx [-L_1, \infty) \times \dots \times [-L_E, \infty)$ so that

$$\int_{-L}^{\infty} dx e^{-zx^2/l_P^2 - wx} = \sqrt{\pi} l_P \exp \left[-\frac{1}{2} \log z + l_P^2 \frac{w^2}{4z} + l_P \frac{e^{-z\bar{L}^2/l_P^2}}{2\sqrt{\pi z \bar{L}}} (1 + O(l_P^2/z\bar{L}^2)) \right],$$

where $\bar{L} = L + l_P^2 \frac{w}{2z}$ and $\text{Re } z = -(\log \mu)''$. The non-analytic terms in \hbar are absent since

$$\lim_{L \rightarrow \infty} e^{-z\bar{L}^2/l_P^2} = 0,$$

for exponentially damped measures.

Perturbative solution

- ▶ For $D_E(L) = \mathbf{R}^E$ and $\mu(L) = \text{const.}$ the perturbative solution is given by the EA diagrams

$$\Gamma_1 = \frac{i}{2} \text{Tr} \log S''_{RC}, \quad \Gamma_2 = \langle S_3^2 G^3 \rangle + \langle S_4 G^2 \rangle,$$

$$\Gamma_3 = \langle S_3^4 G^6 \rangle + \langle S_3^2 S_4 G^5 \rangle + \langle S_3 S_5 G^4 \rangle + \langle S_4^2 G^4 \rangle + \langle S_6 G^3 \rangle, \dots$$

where $G = i(S''_{RC})^{-1}$ is the propagator and $S_n = iS_{RC}^{(n)}/n!$ for $n > 2$, are the vertex weights.

- ▶ When $\mu(L) \neq \text{const.}$, the perturbative solution is given by

$$\Gamma(L) = \bar{S}_{RC}(L) + I_P^2 \bar{\Gamma}_1(L) + I_P^4 \bar{\Gamma}_2(L) + \dots,$$

where

$$\bar{S}_{RC} = S_{RC} - iI_P^2 \log \mu,$$

while $\bar{\Gamma}_n$ is given by the sum of n -loop EA diagrams with \bar{G} propagators and \bar{S}_n vertex weights.

- ▶ Therefore

$$\Gamma_1 = -i \log \mu + \frac{i}{2} \text{Tr} \log S''_{RC}$$

$$\Gamma_2 = \langle S_3^2 G^3 \rangle + \langle S_4 G^2 \rangle + \text{Res}[I_P^{-4} \text{Tr} \log \bar{G}],$$

$$\Gamma_3 = \langle S_3^4 G^6 \rangle + \dots + \langle S_6 G^3 \rangle + \text{Res}[I_P^{-6} \text{Tr} \log \bar{G}] \\ + \text{Res}[I_P^{-6} \langle \bar{S}_3^2 \bar{G}^3 \rangle] + \text{Res}[I_P^{-6} \langle \bar{S}_4 \bar{G}^2 \rangle], \dots$$

- ▶ Since $\log \mu(L) = O((L/L_0)^4)$ and for

$$L_\epsilon > L_c, \quad L_0 > \sqrt{I_P L_c}$$

we get the following large- L asymptotics

$$\Gamma_1(L) = O(L^4/L_0^4) + \log O(L^2/L_c^2) + \log \theta(L) + O(L_c^2/L^2)$$

and

$$\Gamma_{n+1}(L) = O((L_c^2/L^4)^n) + L_{0c}^{-2n} O((L_c^2/L^2)),$$

where $L_{0c} = L_0^2/L_c$.

QG cosmological constant

- ▶ For $L_\epsilon \gg l_P$ and $L_0 \gg \sqrt{l_P L_c}$ the series

$$\sum_n l_P^{2n} \Gamma_n(L)$$

is semiclassical.

- ▶ Let $\Gamma \rightarrow \Gamma/G_N$ so that $S_{eff} = (Re \Gamma \pm Im \Gamma)/G_N$
- ▶ One-loop CC

$$S_{eff} = \frac{S_{Rc}}{G_N} \pm \frac{l_P^2}{G_N L_0^4} V_4 \pm \frac{l_P^2}{2G_N} Tr \log S''_{Rc} + O(l_P^4) \Rightarrow$$

$$\Lambda = \Lambda_c \pm \frac{l_P^2}{2L_0^4} = \Lambda_c + \Lambda_{qg}.$$

- ▶ The one-loop cosmological constant is exact because there are no $O(L^4)$ terms beyond the one-loop order.

- ▶ This is a consequence of the large- L asymptotics

$$\log \bar{S}_{Rc}''(L) = \log O(L^2/\bar{L}_c^2) + \log \theta(L) + O(\bar{L}_c^2/L^2)$$

$$\bar{\Gamma}_{n+1}(L) = O((\bar{L}_c^2/L^4)^n),$$

where $\bar{L}_c^2 = L_c^2 [1 + i l_P^2 (L_c^2/L_0^4)]^{-1/2}$.

- ▶ Note that

$$l_P^2 |\Lambda_{qg}| = \frac{1}{2} \left(\frac{l_P}{L_0} \right)^4 \ll 1,$$

because $L_0 \gg l_P$ is required for the semiclassical approximation.

- ▶ If $\Lambda_c = 0$, the observed value of Λ is obtained for $L_0 \approx 10^{-5} m$ so that $l_P \Lambda \approx 10^{-122}$.

Smooth-spacetime limit

- ▶ Smooth spacetime is described by $T(M)$ with $E \gg 1 \Rightarrow$

$$S_R(L) \approx \frac{1}{2} \int_M d^4x \sqrt{|g|} R(g),$$

$$\Lambda V_4(L) \approx \Lambda \int_M d^4x \sqrt{|g|} = \Lambda V_M,$$

$$\text{Tr} \log S_R''(L) \approx \int_M d^4x \sqrt{|g|} [a(L_K) R^2 + b(L_K) R_{\mu\nu} R^{\mu\nu}],$$

where L_K is defined by

$$L_\epsilon \geq L_K \gg l_P.$$

- ▶ L_K defines a QFT momentum UV cutoff \hbar/L_K . LHC experiments $\Rightarrow L_K < 10^{-19} \text{m} \Leftrightarrow \hbar K > 1 \text{ TeV}$.

Coupling of matter

- ▶ Scalar field on M

$$S_m(g, \phi) = \frac{1}{2} \int_M d^4x \sqrt{|g|} [g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi)] .$$

- ▶ On $T(M)$ we get

$$S_m = \frac{1}{2} \sum_\sigma V_\sigma(L) \sum_{k,l} g_\sigma^{kl}(L) \phi'_k \phi'_l - \frac{1}{2} \sum_\pi V_\pi^*(L) U(\phi_\pi),$$

where $\phi'_k = (\phi_k - \phi_0)/L_0 k$.

- ▶ The total classical action

$$S(L, \phi) = \frac{1}{G_N} S_{Rc}(L) + S_m(L, \phi).$$

Coupling of matter

- ▶ The EA equation

$$e^{i\Gamma(L,\phi)/l_P^2} = \int_{D_E(L)} d^E x \int_{\mathbf{R}^V} d^V \chi \exp \left[i\bar{S}_{Rm}(L+x, \phi+\chi)/l_P^2 - i \sum_{\epsilon} \frac{\partial \Gamma}{\partial L_{\epsilon}} x_{\epsilon}/l_P^2 - i \sum_{\pi} \frac{\partial \Gamma}{\partial \phi_{\pi}} \chi_{\pi}/l_P^2 \right],$$

where $\bar{S}_{Rm} = \bar{S}_{Rc} + G_N S_m(L, \phi)$.

- ▶ Perturbative solution

$$\Gamma(L, \phi) = S(L, \phi) + l_P^2 \Gamma_1(L, \phi) + l_P^4 \Gamma_2(L, \phi) + \dots$$

is semiclassical for $L_{\epsilon} \gg l_P$, $L_0 \gg l_P$ and $|\sqrt{G_N} \phi| \ll 1$. This can be checked in $E = 1$ toy model

$$S(L, \phi) = (L^2 + L^4/L_c^2)\theta(L) + L^2\theta(L)\phi^2(1 + \omega^2 L^2 + \lambda\phi^2 L^2).$$

Coupling of matter

- ▶ $\Gamma(L, \phi) = \Gamma_g(L) + \Gamma_m(L, \phi)$
- ▶ $\Gamma_m(L, \phi) = V_4(L) U_{\text{eff}}(\phi)$ for constant ϕ and $U_{\text{eff}}(0) = 0$.
- ▶ $\Gamma_g(L) = \Gamma_{pg}(L) + \Gamma_{mg}(L)$ and

$$\Gamma_{mg}(L) \approx \Lambda_m V_M + \Omega_m(R, K)$$

in the smooth-manifold approximation and $K = 1/L_K$.

- ▶ $\Omega_m = \Omega_1 l_P^2 + O(l_P^4)$ and

$$\begin{aligned} \Omega_1(R, K) = a_1 K^2 \int_M d^4 x \sqrt{|g|} R + \\ \log(K/\omega) \int_M d^4 x \sqrt{|g|} \left[a_2 R^2 + a_3 R^{\mu\nu} R_{\mu\nu} + a_4 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + a_5 \nabla^2 R \right] \\ + O(L_K^2/L^2). \end{aligned}$$

- ▶ The effective CC

$$\Lambda = \Lambda_c + \Lambda_{qg} + \Lambda_m,$$

where

$$\Lambda_m = \sum_{\gamma} \text{ved}(\gamma, K) = l_P^2 K^4 f(K^2/\omega^2, \lambda l_P^2)$$

is a sum of 1PI vacuum diagrams for $S_m(\eta, \phi)$ with the cutoff K .

- ▶ Postulate

$$\Lambda_c + \Lambda_m = 0,$$

then

$$\Lambda = \Lambda_{qg} = \frac{l_P^2}{2L_0^4} \ll \frac{1}{l_P^2}.$$

- ▶ $\Lambda_c + \Lambda_m = 0$ is possible because it is equivalent to

$$\pm 1/L_c^2 + l_P^2 K^4 f(K^2/\omega^2, \lambda l_P^2) = 0.$$

Conclusions

- ▶ $L_0 \gg l_P$ consistent with the semiclassical approximation.
- ▶ Exact SUSY $\Rightarrow \Lambda_m = 0$ and $\Lambda_c = 0$. If SUSY broken $\Rightarrow \Lambda_m = -\Lambda_c \neq 0$.
- ▶ For small L_ϵ we need a non-perturbative solution of the EA equation. Will be relevant for inflation.
- ▶ Relation to the Wilsonian quantization.
- ▶ EA formalism only applicable for $M = \Sigma \times I$. If

$$M = M_0 \cup (\Sigma \times I), \quad \partial M_0 = \Sigma,$$

then EA can be interpreted as a de-Broglie-Bohm formulation for the Hartle-Hawking wavefunction.

- ▶ Regge SS model represents a simple QG theory consistent with the observations.

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