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Polarization of Photons in Matter–Antimatter Universe

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In this talk we demonstrate the possibility of generation of linear polarization of the electromagnetic field (EMF) due to the quantum effects of the EMF in in matter–antimatter annihilation for anisotropic space of the I type according to Bianchi. It has been established that matter–antimatter annihilation generate linear polarization effects of the EMF in anisotropic Bianchi I type space caused by external gravitational waves. We also ask whether the Universe can be a patchwork consisting of distinct regions of matter and antimatter. We demonstrate that, after recombination, it is impossible to avoid annihilation near regional boundaries. We study the dynamics of this process to estimate two of its signatures: a contribution to the cosmic diffuse γ-ray (CDG) background and a distortion of the cosmic microwave background (CMB) too.

1. Polarization of Photons in Matter–Antimatter Annihilation

For a monochromatic plane EM wave propagating in the z direction,

$$E_x = a_x(t)\cos\left[\omega_0 t - \theta_x(t)\right], \quad E_y = a_y(t)\cos\left[\omega_0 t - \theta_y(t)\right], \tag{1}$$

the Stokes parameters are defined by:

$$I \equiv \langle a_x^2\rangle + \langle a_y^2\rangle, \quad Q \equiv \langle a_x^2\rangle - \langle a_y^2\rangle,$$
$$U \equiv \langle 2a_x a_y \cos(\theta_x - \theta_y)\rangle, \quad V \equiv \langle 2a_x a_y \sin(\theta_x - \theta_y)\rangle, \tag{2}$$

where the brackets $\langle\rangle$ – time averages, I – the average intensity of the radiation, Q and U – linear polarisation, V – circular polarisation.

Q and U are not scalar quantities. If we rotate the reference frame by an angle ϕ around the direction of observation, Q and U transform as:

$$Q' = Q\cos(2\phi) + U\sin(2\phi), \quad U' = -Q\sin(2\phi) + U\cos(2\phi). \tag{3}$$

We can define a polarisation vector $\mathbf{P}$ having:

$$|\mathbf{P}| = \left(Q^2 + U^2\right)^{1/2}, \qquad \alpha = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right). \tag{4}$$

$\mathbf{P}$ is not properly a vector, since it remains identical after a rotation by π around z, thus defining an orientation but not a direction. Q and U can be thought as the components of the second-rank symmetric trace-free tensor:

$$\mathbf{P}_{ab} = \frac{1}{2}\begin{pmatrix} Q & -U\sin\theta \\ -U\sin\theta & -Q\sin^2\theta \end{pmatrix}. \tag{5}$$

1.1. Formalism of the EMF radiation in anisotropic space of the I type according to Bianchi model

Let us consider Maxwell equations for the free the EMF in the metrics

$$ds^2 = dt^2 - \sum_{i=1}^{3} A_i^2(t) \left(dx^i\right)^2 \ , \tag{6}$$

$$\nabla_\mu F^{\mu\nu} = 0 \, , \qquad \nabla_\mu (*F)^{\mu\nu} = 0 \, , \tag{7}$$

where $F_{\mu\nu}$ is electro-magnetic-field tensor and $(*F)^{\mu\nu}$ is adjoint magnitude, defined by the relation $(*F)^{\alpha\beta} = \frac{1}{\sqrt{-g}}[\alpha\beta\gamma\eta]F_{\gamma\eta}$, and $[\alpha,\beta,\gamma,\eta]$ is completely antisymmetric tensor [0123]=1.

The solutions of these equations can be represented in the form of electric- and magnetic-field vectors

$$\mathbf{E}(t,\mathbf{x}) \;=\; \int d^3k e^{i\mathbf{kx}} \left[\mathcal{E}^{\theta}(t,\mathbf{k})\mathbf{e}_{\theta}(t,\mathbf{k}) + \mathcal{E}^{\varphi}(t,\mathbf{k})\mathbf{e}_{\varphi}(t,\mathbf{k})\right],$$

$$\mathbb{H}(t,\mathbf{x}) \;=\; \int d^3k e^{i\mathbf{kx}} \left[\mathcal{H}^{\theta}(t,\mathbf{k})\mathbf{e}_{\theta}(t,\mathbf{k}) + \mathcal{H}^{\varphi}(t,\mathbf{k})\mathbf{e}_{\varphi}(t,\mathbf{k})\right],$$

$$\mathbf{e}_{\theta} \;=\; \cos\theta_t\,\cos\varphi_t\frac{\mathbf{e}_1}{A_1} + \cos\theta_t\,\sin\varphi_t\frac{\mathbf{e}_2}{A_2} - \sin\theta_t\frac{\mathbf{e}_3}{A_3}\,,$$

$$\mathbf{e}_{\varphi} \;=\; -\sin\varphi_t\frac{\mathbf{e}_1}{A_1} + \cos\varphi_t\frac{\mathbf{e}_2}{A_2}\,,$$

$$\mathbf{e}_{k} \;=\; \sin\theta_t\,\cos\varphi_t\frac{\mathbf{e}_1}{A_1} + \sin\theta_t\,\sin\varphi_t\frac{\mathbf{e}_2}{A_2} + \cos\theta_t\frac{\mathbf{e}_3}{A_3}\,,$$

these orthogonal vectors are forming together with the tetrad a unit basis in the momentum space.

The angles θ_t and φ_t are related with the spherical coordinates in the momentum space introduced via the relation

$$(k_1, k_2, k_3) = k\,(\sin\theta\,\cos\varphi,\ \sin\theta\,\sin\varphi,\ \cos\theta)\ ,$$

using the formula

$$(\sin\theta_t\,\cos\varphi_t,\ \sin\theta_t\,\sin\varphi_t,\ \cos\theta_t) = \mu^{-1}\left(\frac{\sin\theta\,\cos\varphi}{A_1},\ \frac{\sin\theta\,\sin\varphi}{A_2}\ ,\ \frac{\cos\theta}{A_3}\right),$$

where

$$\mu^2\ =\ \frac{\sin^2\theta\,\cos^2\varphi}{A_1^2} + \frac{\sin^2\theta\,\sin^2\varphi}{A_2^2} + \frac{\cos^2\theta}{A_3^2}\ .$$

$$
\begin{aligned}
\mathcal{E}^{\theta}(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}}(y^{+}+y^{-})\;, \\
\mathcal{E}^{\varphi}(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}k}\,\frac{d}{dt}(y^{+}-y^{-})\;, \\
\mathcal{H}^{\theta}(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}}(y^{+}-y^{-})\;, \\
\mathcal{H}^{\varphi}(t,\mathbf{k}) &= -\frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}k}\,\frac{d}{dt}(y^{+}+y^{-})\;, \\
b &= \frac{1}{\sqrt{-g}}\left(A_2^2\cos^2\varphi + A_1^2\sin^2\varphi\right)\;,
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\ddot{y}^{r} &- \frac{\dot{b}}{b}\dot{y}^{r} + \left[k^2\mu^2 + rk\Delta\right]y^{r} = 0\;, \\
\Delta &= b\frac{d}{dt}(a/b)\;; \quad a = \frac{\cos\theta\sin 2\varphi}{2\sqrt{-g}}\left(A_2^2 - A_1^2\right)\;.
\end{aligned}
\tag{9}
$$

1.2. Quantum generation of photons in matter–antimatter annihilation

- At t_{in} in the Universe with Friedman metrics there arises the homogeneous anisotropic perturbation in accordance with matter-antimatter annihilation as in Eq. (6).

- At $t < t_{\text{in}}$ the state of the EMF can be described with the density matrix with non-zero occupation number of the photons in the mode $n_0(\nu_0)$ corresponding to the black-body radiation. The latter is strictly constant at $t < t_{\text{in}}$ and constant in the zeroth in respect to the anisotropy parameters approximation at $t > t_{\text{in}}$:

$$\frac{\partial}{\partial t} n_0(\nu_0) = 0.$$

- The frequency ν_0 is equal to the radiation frequency in the current epoch

$$\nu_0 A(t_0) = \nu(t)A(t), \tag{10}$$

where $A(t)$ is the scale factor in the Friedman model at $t < t_{\rm in}$ and $A^3 = (A_1 A_2 A_3)$ at $t > t_{\rm in}$.

- The external gravitational field of the anisotropic Universe brings about the increase of the photon number at $t > t_{\rm in}$

$$n\left(t, \nu_0, \theta, \varphi\right) = n_0(\nu_0) + n_1\left(t, \nu_0, \theta, \varphi\right) + n_q\left(t, \nu_0, \theta, \varphi\right), \tag{11}$$

here $|\delta| \ll 1$ is the correction, describing the anisotropic distribution over momenta which arises due to the anisotropic expansion of the photons already existing to the time moment $t_{\rm in}$

$$n_1\left(t, \nu_0, \theta, \varphi\right) = n_0(\nu_0)\delta\left(t, \nu_0, \theta, \varphi\right), \tag{12}$$

•

$$n_q\left(t,\nu_0,\theta,\varphi\right)=2\left(\sum_{r-\pm1}\left|\beta^r\left(t,\nu_0,\theta,\varphi\right)\right|^2\right)\left(2n_0(\nu_0)+1\right), \qquad (13)$$

it is the additional number of photons which arose due to their generation by matter-antimatter annihilation in the non-stationary gravitational field. β^r is the coefficient of the Bogoliubov transformation at the transition to the time-independent operators of the generation-annihilation of photons bringing to the diagonal form the instantaneous Hamiltonian of the quantised the EMF at the time moment t on the operators of the generation-annihilation, in terms of which the Hamiltonian has the diagonal form at the initial time moment t_{in} ;

$|\beta^r(t,\nu_0,\theta,\varphi)|^2$ is the density of the probability of the generation of a photon with a certain frequency ν_0, direction of the wave vector θ,φ, and the spin projection r on the direction of the wave vector.

- Let us generalise the relations (11) – (13) to the case when the matter of interest is the particle number in the mode, which polarisation vector is oriented along a certain direction in the coordinate frame connected with the wave vector of the photon. Then by analogy to Eq. (4) we can introduce the symbolic vector

$$\mathbf{n}\left(t,\nu_0,\theta,\varphi\right) = \frac{1}{2}\frac{c^3}{h\nu^3(t)}\left(\begin{array}{c} I\left(t,\nu_0,\theta,\varphi\right)+Q\left(t,\nu_0,\theta,\varphi\right) \\ I\left(t,\nu_0,\theta,\varphi\right)-Q\left(t,\nu_0,\theta,\varphi\right) \\ U\left(t,\nu_0,\theta,\varphi\right) \end{array}\right), \quad (14)$$

where I, Q, U are the Stokes parameters of the EM radiation. By analogy with Eq. (11), $\mathbf{n}$ can be represented as

$$\mathbf{n}\left(t,\nu_0,\theta,\varphi\right) = \mathbf{n}_0(\nu_0) + \mathbf{n}_1\left(t,\nu_0,\theta,\varphi\right) + \mathbf{n}_q\left(t,\nu_0,\theta,\varphi\right), \quad (15)$$

where

- $$\mathbf{n}_0(\nu_0) = n_0(\nu_0) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},$$

  which corresponds to the isotropic nonpolarised radiation. In the case when there is no scattering of the photons on the electrons of the cosmic plasma, $\mathbf{n}_1$ can be represented as

$$\mathbf{n}_1(t,\nu_0,\theta,\varphi) \;=\; n_0(\nu_0)\left(\alpha(t,\nu_0)\mathbf{a}(\theta,\varphi) + \bar{\alpha}(t,\nu_0)\bar{\mathbf{a}}(\theta,\varphi)\right), \tag{16}$$

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\left(\cos^2\theta - \frac{1}{3}\right), \quad \bar{\mathbf{a}} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\left(1 - \cos^2\theta\right)\cos 2\varphi.$$

This corresponds to the start of the dependence on angles, i.e. anisotropy, in the distribution of the photons over momenta. The coefficients α and $\bar{\alpha}$ characterise the degree of the quadrupole anisotropy of the CMBR.

- The quantity $\mathbf{n}_q$ describes the contribution of the quantum effects in matter-antimatter annihilation process. In the linear with respect to the anisotropy parameters approximation of the metrics (6) it can be represented in the form:

$$\mathbf{n}_q\left(t, \nu_0, \theta, \varphi\right) = \frac{2n\alpha(\nu_0)+1}{2}\frac{c^3}{h\nu^3(t)}\left(\begin{array}{c} Q^{\mathrm{ann}}\left(t, \nu_0, \theta, \varphi\right) \\ -Q^{\mathrm{ann}}\left(t, \nu_0, \theta, \varphi\right) \\ 2U^{\mathrm{ann}}\left(t, \nu_0, \theta, \varphi\right) \end{array}\right). \quad (17)$$

The quantities I^{ann}, Q^{ann}, U^{ann} are the annihilation Stokes parameters (ASP), evaluated for the case when the initial state of the EMF at the time of the matter-antimatter annihilation t_{in} was only the CMB radiation.

2. Polarization of Photons due to the quantum effects in matter–antimatter Universe

The Stokes parameters are :

$$J^{ab\,ann} = \frac{1}{2}\begin{pmatrix} I^{\rm ann}+Q^{\rm ann} & U^{\rm ann}-iV^{\rm ann} \\ U^{\rm ann}+iV^{\rm ann} & I^{\rm ann}-Q^{\rm ann} \end{pmatrix}, \qquad (18)$$

where $J^{ab\,ann} = J_{+}^{ab\,ann} + J_{-}^{ab\,ann}$,

$$J_{\pm}^{ab\,ann}(t,\mathbf{k}) \;=\; \frac{1}{2}\,\frac{hk}{\mu(t,\theta,\varphi)}\,\langle 0_{t_{\rm in}}|N_t\left[\hat{\mathcal{E}}^a(t,\mathbf{x},\mathbf{k}),\hat{\mathcal{E}}^b(t,\mathbf{x},\mathbf{k})\right]|0_{t_{\rm in}}\rangle.$$

Here $\hat{\mathcal{E}}^a(t,\mathbf{x},\mathbf{k})$ are the components of the spectral component of the vector of electric field (8), multiplied by $\exp(i\mathbf{kx})$.

We have shown that $J^{ab\,ann} = 0$, i.e. the generating photons do not have an admixture of the circular polarisation.

The symmetric part of the polarisation tensor could be expressed via spectral components of the averages of the operator of energy-momentum tensor

$$T_{\mu\nu}^{\rm ann}(t) \;=\; \int d\varphi d\theta \sin\theta \int dK_0\,(t,k,\theta,\varphi)\;\widetilde{T}_{\mu\nu}^{\rm ann}\,(t,k,\theta,\varphi)\,,$$

$$K_0\,(t,k,\theta,\varphi) \;=\; ck\mu\,(t,\theta,\varphi)\;\;,$$

as follows

$$J_{+}^{ab} = G_{\mu\nu}^{ab}\widetilde{T}_{\mu\nu}^{\rm ann}.$$

So the APC can be presented as follows

$$\left(I^{\mathrm{ann}}, Q^{\mathrm{ann}}, U^{\mathrm{ann}}, V^{\mathrm{ann}}\right) = \frac{hk^3}{V} \sum_{r=\pm 1} \left(2s^r, u^r, r\tau^r, 0\right), \tag{19}$$

where the functions s^r, u^r, τ^r satisfy the set of the equations

$$\left\{ \begin{array}{l} \dot{s}^r = \frac{W}{2} u^r + r\frac{\overline{W}}{2}\tau^r, \\ \dot{u}^r = W\left(2s^r + 1\right) - \left(r\overline{W} + 2cK_0\right)\tau^r, \\ \dot{\tau}^r = r\overline{W}\left(2s^r + 1\right) + \left(r\overline{W} + 2cK_0\right)u^r, \end{array} \right. \tag{20}$$

$$\left(u^r\right)^2 + \left(\tau^r\right)^2 = 4s^r\left(s^r + 1\right)$$

with the initial values $s^r(t_{\mathrm{in}}) = u^r(t_{\mathrm{in}}) = \tau^r(t_{\mathrm{in}}) = 0$.

The quantities W and $\overline{W}$ in the metrics, linear in respect to the anisotropy parameters of metrics (6) are as follows:

$$
\begin{aligned}
W &= \left(1-\cos^2\theta\right)\Delta H + \frac{\overline{\Delta H}}{2}\left(1+\cos^2\theta\right)\cos 2\varphi, \\
\overline{W} &= -\cos\theta\,\sin 2\varphi\overline{\Delta H},
\end{aligned}
\tag{21}
$$

where

$$
\begin{aligned}
\Delta H &= H - \frac{1}{2}\left(H_1 + H_2\right), \quad \overline{\Delta H} = H_1 - H_2, \\
H^3 &= (H_1 H_2 H_3), \quad H_i = \dot{A}_i / A_i
\end{aligned}
$$

(the parameters ΔA, $\overline{\Delta A}$, A could be introduced similarly).

The set of the equations (20) plays the part of the transfer equations for ASP.

Let us analyse the expressions (19), solving (20) via expansion of the functions in a power series in respect to small parameter $\tilde{h}$ which is introduced as

$$\Delta H \to \tilde{h}\Delta H, \quad \overline{\Delta H} \to \tilde{h}\overline{\Delta H}, \qquad \Delta A \to \tilde{h}\Delta A, \quad \overline{\Delta A} \to \tilde{h}\overline{\Delta A}.$$

In doing so,

$$s^r = \sum_{n=0} \tilde{h}^n s_n^r\,, \quad u^r = \sum_{n=0} \tilde{h}^n u_n^r\,, \quad \tau^r = \sum_{n=0} \tilde{h}^n \tau_n^r\,. \tag{22}$$

The expansions of W and $\overline{W}$ are given with (21). Further we shall keep in the expansion (22) only linear terms in respect to $\tilde{h}$. In the zeroth approximation in respect to $\tilde{h}$ it follows from (20), (21), taking into the account the initial values, that

$$s_0^r = u_0^r = \tau_0^r = 0.$$

This means that there are no photon quantum effects in isotropic case. In the linear approximation the set of the equations for s_1^r, u_1^r, τ_1^r is

$$\dot{s}_1^r = 0, \quad \dot{u}_1^r = W - 2\nu\tau_1^r, \quad \dot{\tau}_1^r = r\overline{W} + 2\nu u_1^r$$

(in the zeroth approximation in respect to $\tilde{h}$ $K_0(t,k,\theta,\varphi) = k_0/A(t) \equiv \nu(t)$). Solving this set, one can obtain the expressions for ASP in the linear approximation:

$$(I^{\mathrm{ann}},\, Q^{\mathrm{ann}},\, U^{\mathrm{ann}}) = \tag{23}$$

$$\frac{2h\nu^3}{c^3}\int\limits_{t_{\mathrm{in}}}^{t} \left(0, W(t'), \overline{W}(t')\right)\cos\left(2(\Omega(t)-\Omega(t'))\right)dt',$$

$$\Omega(t) = \int dt\nu(t).$$

Such a distribution of the ASP quantum effects in the anisotropic gravitational field is unusual from the viewpoint of the classical electrodynamics. The reason is the structure of the vacuum energy-momentum tensor of the EMF in the external gravitational field. $Zeldovich$ and $Starobinsky$ have remarked that quantum effects of the material field in the external anisotropic gravitational field bring about the breaking of the condition of the energy dominance of the energy-momentum tensor (EMT) of these fields.) This fact shows itself in different dependence of the EMT components on the anisotropy parameters of the metrics (6), namely:

$$T_0^0 \sim \tilde{h}^2, \qquad T^{ik} \sim \tilde{h}, \qquad i, k = 1, 2, 3.$$

The fact that EMT does not satisfy the condition of the energy dominance means that it contains both the contribution from the really generated particles and the contribution of the EMF in matter-antimatter annihilation process.

Let us bring $\mathbf{n}_q$ to the form analogous that of $\mathbf{n}_1$, singling out explicitly the dependence on the angles θ and φ. To this end we use (21) and (23), then

$$\mathbf{n}_q(t,\nu_0,\theta,\varphi) = \frac{2n_0(\nu_0)+1}{2}\left(\beta_q(t,\nu_0)\mathbf{b}(\theta,\varphi)+\bar{\beta}_q(t,\nu_0)\bar{\mathbf{b}}(\theta,\varphi)\right), \quad (24)$$

$$\beta_q(t) = \int\limits_{t_{\rm in}}^{t} \Delta H(t')\cos\left(2(\Omega(t)-\Omega(t'))\right)dt'. \quad (25)$$

The expression for $\bar{\beta}_q$ can be obtained from (25), substituting $\overline{\Delta H}$ for ΔH.

Vectors $\mathbf{b}$ and $\bar{\mathbf{b}}$ are defined via the relations (analogously as was defined by Basco and Polnarev):

$$\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} (1 - \cos^2\theta)\,, \quad \bar{\mathbf{b}} = \frac{1}{2} \begin{pmatrix} (1 + \cos^2\theta)\cos 2\varphi \\ -(1 + \cos^2\theta)\cos 2\varphi \\ 4\cos\theta\, \sin 2\varphi \end{pmatrix}.$$

Assuming that $n_0 \gg 1$ and substituting (16), (24) into (15), we obtain

$$\mathbf{n} = \mathbf{n}_0 + n_0 \left[\alpha\mathbf{a} + \beta_q\mathbf{b} + \bar{\alpha}\bar{\mathbf{a}} + \bar{\beta}_q\bar{\mathbf{b}}\right]. \qquad (26)$$

The quantities β_q and $\bar{\beta}_q$ are related with the degree of the linear polarisation of the EMF in matter-antimatter annihilation. The relation (26) is similar to that derived by Basco under the solution the radiation transfer equation taking into the account Thomson scattering of the photons on the electrons of the cosmic plasma.

The quantities which undergo the measurement in the course of experiment are the Stokes parameters I, Q and U. Let us evaluate, for example, the Stokes parameter Q in the Heckman-Schüking model. According to Eqs. (14) and (26) we have

$$Q(t,\nu_0,\theta,\varphi) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \left(1-\cos^2\theta\right) \beta_q(t,\nu_0).$$

The dependence of Q on the time t is determined with the quantity β_q (25). Let us transform it to more clear form. To this end let us temporarily change in the integral (25) the integration variable X as follows:

$$\begin{aligned} X &= (1+\chi)^{-1/2}, \quad \Delta H = \Delta H_0 (1+\chi)^3 = \Delta H_0 / X^6, \\ dt &= -\frac{1}{H_0}\frac{d\chi}{(1+\chi)^{5/2}} = \frac{2}{H_0} X^2 dX\ , \quad \nu(t) = \nu_0(1+\chi) = \nu_0 X^{-2}, \end{aligned}$$

where ΔH_0, H_0, ν_0 are the current values of the corresponding parameters.

By this change of variable the integral is brought to the form

$$\beta_q = \frac{\Delta H_0}{H_0} \int\limits_{X_{\rm in}}^{X} \frac{\cos\lambda(X - X')}{X'^4} dX', \qquad (27)$$

where $\lambda = 4\nu_0/H_0 \approx 10^{30}$ is a large parameter. Estimating (27) asymptotically in λ, we obtain

$$\beta_q = \frac{\Delta H_0}{H_0} \frac{1}{\lambda X_{\rm in}^4} \sin\left(\lambda(X - X_{\rm in})\right). \qquad (28)$$

Let us change in the last formula from the current variable X to the synchronous time t according to the relation $X = (3H_0t/2)^{1/3}$. The time t is synchronous cosmological time, counted from the singularity.

Let us divide it into two summands as follows: $t = t_0 + t'$, $\qquad t' \ll t_0$, where $t_0 = \frac{2}{3H_0}$ is the time counted from the beginning of the expansion, corresponding to the current epoch, and t' is the current time, for example, the period, during which the observations take place. Then

$$Q(t, \nu_0, \theta, \varphi) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0)(1 - \cos^2\theta) \frac{\Delta H_0}{H_0} \frac{\chi_{\rm in}}{\lambda} \sin\left(2\nu_0 t' + \lambda(1 - X_{\rm in})\right).$$

Let us discuss the possibility of experimental measuring of Q. The power of the polarised component of interest for us of the EMF radiation, hitting the aerial of the radio-telescope, with the directivity diagram $P_n(\theta, \varphi)$ and the effective area of the surface $A_{\rm eff}$ per unit frequency band is defined as follows

$$W(t) = \frac{1}{2} A_{\rm eff} \left| \int\!\!\int_{\Omega} Q(t, \nu_0, \theta, \varphi) P_n(\theta, \varphi) d\Omega \right|. \qquad (29)$$

Separating the dependence on angle, we bring (29) to the form

$$W(t) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \Omega_{\mathrm{eff}}^Q \left| \beta_q(t, \nu_0) \right| .$$

The quantity $W(t)$ is the instantaneous power of the signal in the aerial. On the load of the aerial the voltage is induced, the square of which is proportional to the instantaneous power, so that the current at the output of the detector is as follows

$$i_{\mathrm{det}}^Q(t) = k' \hat{U}^2(t) = k' W(t) = k' \Omega_{\mathrm{eff}}^Q \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \left| \beta_q(t, \nu_0) \right| .$$

After averaging over time, we obtain

$$\overline{i^{Q}_{\rm det}} = k' \left(\Omega^{Q}_{\rm eff}\right) \left(\frac{2h\nu_0^3}{c^3} n_0(\nu_0)\right) \frac{\Delta H_0}{H_0} \frac{\chi^2_{\rm in}}{\lambda} \frac{2}{\pi} \, .$$

Similarly, measuring the Stokes parameter I in the zeroth approximation with respect to the anisotropy parameter ΔH, we arrive at

$$\overline{i^{I}_{\rm det}} = k' \left(\Omega^{T}_{\rm eff}\right) \left(\frac{2h\nu_0^3}{c^3} n_0(\nu_0)\right) .$$

The observable degree of polarisation turns out to be

$$P = \frac{\overline{i^{Q}_{\rm det}}}{\overline{i^{I}_{\rm det}}} = \frac{2}{\pi} \frac{\Omega^{Q}_{\rm eff}}{\Omega^{I}_{\rm eff}} \frac{\Delta H_0}{H_0} \frac{\chi^2_{\rm in}}{\lambda} \, . \tag{30}$$

3. Distortion of the CMB

Measurements of the CMB, being much more precise than those of the CDG, might be expected to provide the most stringent constraint on the $B = 0$ universe. In this section, we use A. G. Cohen calculation of the annihilation rate to estimate the distortion of the CMB spectrum. In performing this calculation, A. G. Cohen et al. made several approximations that somewhat overestimate the effect. Nonetheless, the consequent distortion lies well below the observed limit, and provides no constraint at all.

Annihilation produces relativistic electrons and energetic photons. Annihilation electrons have a direct effect on the CMB by scattering photons to higher energies, thereby skewing the CMB spectrum. Moreover these electrons heat the ambient plasma. The heated plasma produces an additional indirect spectral distortion. (The energetic photons from neutral pion decay have energies too high to have much effect on the CMB.)

To compute the direct effect, we must determine the number of CMB photons scattered from energy ω_i to ω_f by a single electron. This function, $d^2N(\omega_f, \omega_i)/d\omega_f\, d\omega_i$, is computed by A. G. Cohen. The electron multiplicity per $p\bar{p}$ annihilation is similar to the photon multiplicity, measured in the experiment to be $\bar{g} \simeq 3.8$. The number of annihilation electrons made per unit volume and time is $\bar{g}\, J/d$, where $1/d \equiv y/d_0$ is the average domain surface-to-volume ratio at epoch y. The spectral distortion $\delta u_\gamma(\omega)$ (energy per unit volume and energy) satisfies a transport equation

$$\left(y\,\frac{\partial}{\partial y} + \omega\,\frac{\partial}{\partial \omega} - 3 \right)\, \delta u_\gamma(\omega, y) =$$

$$\frac{\omega \bar{g}\, J(y)}{H(y)\, d(y)} \int d\nu \left(\frac{d^2N(\nu,\omega)}{d\nu d\omega} - \frac{d^2N(\omega,\nu)}{d\nu d\omega} \right) \quad \equiv A(\omega, y)\ . \qquad (31)$$

We have ignored absorption of UV photons by neutral hydrogen because the $B = 0$ universe is largely ionized.

The direct contribution to the CMB distortion is the solution to Eq. (31) evaluated at the current epoch: $\delta u_\gamma(\omega) \equiv \delta u_\gamma(\omega, 1)$. It is given by

$$\delta u_\gamma(\omega) = \int_{y_R}^{y_S} \frac{dy}{y^4} A(\omega\, y, y) \ , \tag{32}$$

where it has confined the source to $1100 > y > 20$, the era of unavoidable annihilation. To evaluate the integral we use the annihilation rate J computed by Cohen. Where it displays the result for a current domain size of 20 Mpc. Note that $|\delta u_\gamma(\omega)|$ is always less than 3×10^{-3} cm^{-3} $\simeq$ $1.8 \times 10^{-6}\ T_0^3$. The limit set by COBE–FIRAS on rms departures from a thermal spectrum is $|\delta u_\gamma(\omega)| < 7.2 \times 10^{-6}\, T_0^3$ throughout the energy range $T_0 < \omega < 10\ T_0$. This upper limit is four times larger than computed signal given by A. G. Cohen for the minimum domain size. Because larger domains yield proportionally smaller results, we have no constraint on the $B = 0$ universe.

The indirect contribution to the CMB distortion results from a temperature difference $T - T_\gamma$ between the heated ambient fluid and the CMB. It may be described by the Sunyaev–Zeldovich parameter Y

$$Y = \int \frac{\sigma_T \, n_e (T - T_\gamma)}{m_e c^2} \, dl \ , \tag{33}$$

where the integral is along the photon path $dl = -c \, dy / y \, H(y)$.

To compute Y, A. G. Cohen et al. used the higher temperature profile. Their result is $Y \lesssim 9 \times 10^{-7}$, which is over an order of magnitude below the COBE–FIRAS limit of $|Y| < 1.5 \times 10^{-5}$. We conclude[1] that current observations of the CMB spectrum yield no constraint on the $B = 0$ Universe.

[1] An additional contribution to Y arises as CMB photons pass through transitional regions being re-ionized, but is two orders of magnitude smaller than the effect we discussed.

4. The Diffuse Gamma-Ray Spectrum

In this section, we use A. G. Cohen et al. conservative calculation of the annihilation rate to determine a lower bound to the CDG signal. They obtained that annihilation in a $B = 0$ universe produces far more γ-rays than are observed.

The relic spectrum of γ-rays consists primarily of photons from π^0 decay. Let $\Phi(E)$ denote the inclusive photon spectrum in $p\bar{p}$ annihilation, normalized to $\bar{g}$, the mean photon multiplicity[2]. The average number of photons made per unit volume, time and energy is $\Phi(E)\, J/d$.

[2] The measured photon spectrum can be found in the experiment .

These photons scatter and redshift, leading to a spectral flux of annihilation photons $F(E,y)$ (number per unit time, area, energy and steradian) satisfying the transport equation:

$$\left(y\frac{\partial}{\partial y}+E\frac{\partial}{\partial E}-2\right)F(E,y)=-\frac{1}{H(y)}\,\Phi(E)\,\frac{c\,J}{4\,\pi\,d}+R(E,y)\,. \tag{34}$$

The first term on the RHS is the annihilation source and the second is a scattering sink. If we slightly underestimate $F(E,y)$ by treating all scattered photons as effectively absorbed. In this case:

$$R(E,y)=\frac{c\,\sigma_\gamma(E)\,n_e(y)}{H(y)}\,F(E,y)\equiv g(E,y)\,F(E,y)\,, \tag{35}$$

with σ_γ the photon interaction cross section and $n_e(y)$ the electron density. For the relevant photon energies, it matters little whether photons encounter bound or unbound electrons.

Integration of Eqs. (34)–(35) gives the photon flux today, $F(E) \equiv F(E, 1)$:

$$F(E) = \int_{y_S}^{y_R} \frac{c\, J(y')\, \Phi(Ey')}{4\pi\, d(y')} \exp\left[- \int_1^{y'} \frac{dy''}{y''}\, g(Ey'', y'') \right] \frac{dy'}{H(y')\, y'^3} \,. \tag{36}$$

This conservative lower limit to the γ-ray signal conflicts with observations by several orders of magnitude and over a wide range of energies, for all values of $d_0 \lesssim 10^3$ Mpc, comparable to the size of the universe. It could be argued that the satellite data excludes even larger domain sizes, but it would be soon run into questions of the precise geometry and location of these nearly horizon-sized domains.

5 Conclusions

- The Universe contains lots of light (some 400 microwave background photons per cc), a little matter (a few baryons and electrons for every billion photons) and practically no antimatter, at least in our neighbourhood.

- Various balloon- and satellite-borne detectors have observed cosmic-ray positrons and antiprotons. Their flux is compatible with the expectation for the secondary products of conventional (matter) cosmic rays impinging on interstellar matter (gas and dust).

- The $\bar{p}/p$ ratio is expected to diminish precipitously below a kinetic energy of a few GeV: at the high energy required to produce these secondaries, the production of a slow $\bar{p}$ is unlikely.

- The cosmic-ray flux of many different nuclei is well measured in a domain of kinetic energy (per nucleon) extending from a few MeV to circa 1 TeV. But for a small fraction of anti-deuterons, one does not expect an observable flux of antinuclei, for the energy required to make these fragile objects in matter–antimatter collisions is far in excess of their binding energy. No convincing observation of $Z > 2$ antinuclei has been reported.

- It is often emphasized that the observation of a single antinucleus would be decisive evidence for an antimatter component of the Universe: it is likely that $\overline{\mathrm{He}}$ would be the result of primordial antinucleosynthesis; $\overline{\mathrm{C}}$ would presumably originate in an antistar.

- The "photonic" astronomer cannot determine whether or not another galaxy is made of matter or of antimatter.

- But galaxies in collision are often observed. An encounter involving a galaxy and an antigalaxy would be spectacular.

- Galaxies are supposed to have undergone a phase of recollapse onto themselves, after they lagged behind the general Hubble expansion to become objects of fixed size, at a redshift of a few. This recollapse is reckoned to mix and virialize the galactic material, and to re-ionize its ordinary matter. If this process could occur in a galaxy containing matter and antimatter it would annihilate the minority ingredient, or blow the galaxy apart.

- Nonetheless, the search for ordinary or neutron antistars is of interest, for they may not "belong" to our galaxy, but be intruders from afar. These objects would accrete interstellar gas and shine γ rays.

- The background of this paper constitutes the idea, which attempts to explain some linear polarization of the EMF in the Universe as produced by quantum effects in matter-antimatter annihilation process. The peculiar feature of Eq. (29) is that β_q parametrically depends on the time moment $t_{\rm in}$ when the anisotropic perturbations have arisen on the initially isotropic space-time background of the Universe.

- The importance of the EMF polarisation concerning the physics of matter-antimatter annihilation in the early Universe relies on the fact that scalar fluctuations and the photon quantum effects can produce only linear polarisation and no circular polarisation. A measurement of the linear polarisation could therefore be interpreted as the detection of gravitational waves caused by matter-antimatter annihilation.

- Thus, it turns out that in the case of the transparent medium when there is no scattering of the photons, the EMF radiation in the homogeneous anisotropic and non-stationary Universe becomes linearly polarised due to the quantum effects of the photon generation by matter-antimatter annihilation.

- It is remarkable that the angular dependence of the photon number is quadrupole and completely coincides with that arising under the scattering of the CMB photons on electrons in the epoch of the recombination or the secondary ionization.