A New Venue of Spontaneous Supersymmetry Breaking in Supergravity

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Background material and further development of:

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- and ideas from some older works:
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Introduction - Overview of Talk

- We present a qualitatively new mechanism for dynamical spontaneous breakdown of supersymmetry. Specifically, we construct a modified formulation of standard minimal $N = 1$ supergravity as well as anti-de Sitter supergravity.
- The modification is based on an idea worked out in detail in previous publications by some of us, where a new formulation of (non-supersymmetric) gravity theories was proposed employing an ***alternative volume form*** (volume element, or generally-covariant integration measure) in the pertinent Lagrangian action. The latter is defined in terms of auxiliary (pure-gauge) fields instead of the standard Riemannian metric volume form.
- Invariance under supersymmetry is preserved due to addition of compensating antisymmetric tensor gauge field.

Introduction - Overview of Talk

- The new supergravity formalism naturally triggers the appearance of a ***dynamically generated cosmological constant*** as an arbitrary integration constant which signifies ***spontaneous (dynamical) breaking of supersymmetry***.
- Applying the new formalism to anti-de Sitter supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a ***very small effective observable cosmological constant*** as well as a ***large gravitino mass*** as required by modern cosmological scenarios for slowly expanding universe of today.

Introduction - Why Supersymmetry

- ***Supersymmetry*** – fundamental extended space-time symmetry of Nature at ultra-high energies unifying bosons (integer-spin particles) and fermions (half-integer spin particles).
- Theoretical highlights: drastically reducing the number of apriori independent physical parameters; drastically reducing (in some cases even eliminating) ultraviolet divergencies in quantum field theory; possible solution of the *hirarchy/fine-tuning problems*

Introduction - Why Supersymmetry

Apart from elementary particle physics, **supersummetry** plays an increasingly active role both as conceptional paradigm as well as important mathematical tool in various other areas of modern theoretical physics and mathematics:

- extensions of general relativity and cosmology;
- condensed matter and nuclear physics;
- theory of integrable systems – solitons.

Introduction - Supersymmetry Breaking

Unfortunately, supersymmetry is *not an exact* symmetry of Nature. Otherwise, we would observe bosonic (integer-spin) counterparts with the same masses of the fermionic particles – protons, electrons, *etc.*. Therefore, supersymmetry must be **spontaneously broken**.

- Spontaneous symmetry breakdown – symmetry-generating charges are conserved, however, the ground state ("vacuum") is *not* invariant.
- Spontaneous symmetry breakdown is always accompanied by the appearance of certain mass (energy) scale of the breakdown.

Introduction - Supersymmetry Breaking

- Typically, the scale of spontaneous symmetry breaking is generated by the appearance of non-zero vacuum expectation values of (quantum) fields non-trivially transforming under the pertinent symmetry group. An important example is the ***Brout-Englert-Higgs*** mechanism in the Standard Model of particle physics.
- In supergravity (supersymmetric generalizations of ordinary Einstein general relativity) there is another way to spontaneously break supersymmetry – via ***dynamical generation of non-zero cosmological constant***.

In what follows we will present an explicit realization of the above mechanism.

Two-Measure Theories

In a series of previous papers [E.Guendelman *et.al.*] a new class of generally-covariant (non-supersymmetric) field theory models including gravity – called “two-measure theories” (TMT) was proposed.

- TMT appear to be promising candidates for resolution of various problems in modern cosmology: the *dark energy* and *dark matter* problems, the fifth force problem, etc.
- Principal idea – employ an alternative volume form (volume element or generally-covariant integration measure) on the space-time manifold in the pertinent Lagrangian action.

Two-Measure Theories

The alternative volume element is given by the following ***non-Riemannian*** integration measure density:

$$\Phi(B) \equiv \frac{1}{(D-1)!}\varepsilon^{\mu_1\ldots\mu_D}\,\partial_{\mu_1}B_{\mu_2\ldots\mu_D}\;, \tag{1}$$

where $B_{\mu_1\ldots\mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor gauge field, which can also be parametrized in terms of D scalar fields $B_{\mu_1\ldots\mu_{D-1}} = \frac{1}{D}\varepsilon_{IJ_1\ldots J_{D-1}}\phi^I\partial_{\mu_1}\phi^{J_1}\ldots\partial_{\mu_{D-1}}\phi^{J_{D-1}}$, so that: $\Phi(B) = \frac{1}{D!}\varepsilon^{\mu_1\ldots\mu_D}\,\varepsilon_{I_1\ldots I_D}\partial_{\mu_1}\phi^{I_1}\ldots\partial_{\mu_D}\phi^{I_D}$.

Recall: the standard Riemannian integration measure density is $\sqrt{-g}$, where $g \equiv \det\|g_{\mu\nu}\|$ is the determinant of the corresponding Riemannian metric $g_{\mu\nu}$.

Two-Measure Theories

To illustrate the TMT formalism let us consider the following action:

$$S = c_1 \int d^D x\, \Phi(B) \Big[L^{(1)} + \frac{\varepsilon^{\mu_1 \ldots \mu_D}}{(D-1)!\sqrt{-g}} \partial_{\mu_1} H_{\mu_2 \ldots \mu_D} \Big] + c_2 \int d^D x \sqrt{-g}\, L^{(2)} \qquad (2)$$

with the following notations:

- The Lagrangians $L^{(1,2)} \equiv \frac{1}{2\kappa^2} R + L^{(1,2)}_{\rm matter}$ include both standard Einstein-Hilbert gravity action as well as matter/gauge-field parts. Here $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ is the scalar curvature within the first-order (Palatini) formalism and $R_{\mu\nu}(\Gamma)$ is the Ricci tensor in terms of the independent affine connection $\Gamma^{\mu}_{\lambda\nu}$.

Two-Measure Theories

- In general, second Lagrangian $L^{(2)}$ might contain also higher curvature terms like R^2.
- In the first ***modified-measure term*** of the action (2) we have included an additional term containing another auxiliary rank $(D-1)$ antisymmetric tensor gauge field $H_{\mu_1\ldots\mu_{D-1}}$. Such term would be purely topological (total divergence) one if included in standard Riemannian integration measure action like the second term with $L^{(2)}$ on the r.h.s. of (2).

$H_{\mu_1\ldots\mu_{D-1}}$ similarly will turn out to be pure-gauge degree of freedom, however, both auxiliary tensor gauge fields (B and H) will nevertheless play crucial role in the sequel.

Two-Measure Theories

Varying (2) w.r.t. H and B tensor gauge fields we get:

$$\partial_\mu\Big(\frac{\Phi(B)}{\sqrt{-g}}\Big) = 0 \quad \to \quad \frac{\Phi(B)}{\sqrt{-g}} \equiv \chi = \text{const} \;, \tag{3}$$

$$L^{(1)} + \frac{\varepsilon^{\mu_1 \ldots \mu_D}}{(D-1)!\sqrt{-g}} \partial_{\mu_1} H_{\mu_2 \ldots \mu_D} = M = \text{const} \;, \tag{4}$$

where χ (ratio of the two measure densities) and M are ***arbitrary integration constants***.

Performing canonical Hamiltonian analysis of (2) we find that the above integration constants M and χ are in fact ***constrained a'la Dirac canonical momenta*** of B and H.

Two-Measure Theories

- Now, varying (2) w.r.t. $g^{\mu\nu}$ and taking into account (3)–(4) we arrive at the following effective Einstein equations (in the first-order formalism):

$$R_{\mu\nu}(\Gamma) - \frac{1}{2} g_{\mu\nu} R + \Lambda_{\rm eff} g_{\mu\nu} = \kappa^2 T^{\rm eff}_{\mu\nu} , \tag{5}$$

with effective energy-momentum tensor:

$$T^{\rm eff}_{\mu\nu} = g_{\mu\nu} L^{\rm eff}_{\rm matter} - 2 \frac{\partial L^{\rm eff}_{\rm matter}}{\partial g^{\mu\nu}} \;\; , \;\; L^{\rm eff}_{\rm matter} \equiv \frac{1}{c_1 \chi + c_2} \Bigl[c_1 L^{(1)}_{\rm matter} + c_2 L^{(2)}_{\rm matter} \Bigr] \tag{6}$$

and with a ***dynamically generated effective cosmological constant*** thanks to the non-zero integration constants

$$\Lambda_{\rm eff} = \kappa^2 \left(c_1 \chi + c_2 \right)^{-1} \chi M \; . \tag{7}$$

Supersymmetric Higgs Effect in Supergravity

- Let us now apply the above TMT formalism to construct a modified-measure version of $N=1$ supergravity in $D=4$. Recall the standard component-field action of $D=4$ (minimal) $N=1$ supergravity:

$$S_{\rm SG} = \frac{1}{2\kappa^2}\int d^4x\, e\Big[R(\omega,e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda\Big]\;, \tag{8}$$

$$e = \det\|e^a_\mu\| \;\;,\;\; R(\omega,e) = e^{a\mu}e^{b\nu}R_{ab\mu\nu}(\omega)\;. \tag{9}$$

$$R_{ab\mu\nu}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega^c_{\mu a}\omega_{\nu cb} - \omega^c_{\nu a}\omega_{\mu cb}\;. \tag{10}$$

$$D_\nu \psi_\lambda = \partial_\nu \psi_\lambda + \frac{1}{4}\omega_{\nu ab}\gamma^{ab}\psi_\lambda \;\;,\;\; \gamma^{\mu\nu\lambda} = e^\mu_a e^\nu_b e^\lambda_c \gamma^{abc}\;, \tag{11}$$

where all objects belong to the first-order "vierbein" (frame-bundle) formalism.

Supersymmetric Higgs Effect in Supergravity

The vierbeins e^a_μ (describing the graviton) and the spin-connection $\omega_{\mu ab}$ ($SO(1,3)$ gauge field acting on the gravitino ψ_μ) are *a priori* independent fields (their relation arises subsequently on-shell); $\gamma^{ab} \equiv \frac{1}{2}\left(\gamma^a\gamma^b - \gamma^b\gamma^a\right)$ *etc.* with γ^a denoting the ordinary Dirac gamma-matrices. The invariance of the action (8) under local supersymmetry transformations:

$$\delta_\epsilon e^a_\mu = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu \quad , \quad \delta_\epsilon\psi_\mu = D_\mu\varepsilon \tag{12}$$

follows from the invariance of the pertinent Lagrangian density up to a total derivative:

$$\delta_\epsilon\Big(e\big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda\big]\Big) = \partial_\mu\big[e\big(\bar{\varepsilon}\zeta^\mu\big)\big] \ , \tag{13}$$

where ζ^μ functionally depends on the gravitino field ψ_μ.

Supersymmetric Higgs Effect in Supergravity

We now propose a modification of (8) by replacing the standard generally-covariant measure density $e = \sqrt{-g}$ by the alternative measure density $\Phi(B)$ (Eq.(1) for $D = 4$):

$$\Phi(B) \equiv \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda} \ , \tag{14}$$

and we will use the general framework described above. The modified supergravity action reads:

$$S_{\rm mSG} = \frac{1}{2\kappa^2} \int d^4x \, \Phi(B) \left[R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\, e} \partial_\mu H_{\nu\kappa\lambda} \right] , \tag{15}$$

where a new term containing the field-strength of a 3-index antisymmetric tensor gauge field $H_{\nu\kappa\lambda}$ has been added.

The equations of motion w.r.t. $H_{\nu\kappa\lambda}$ and $B_{\nu\kappa\lambda}$ yield:

$$\partial_\mu\Big(\frac{\Phi(B)}{e}\Big) = 0 \quad \rightarrow \quad \frac{\Phi(B)}{e} \equiv \chi = \text{const} \ , \tag{16}$$

$$R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\, e} \partial_\mu H_{\nu\kappa\lambda} = 2M \ , \tag{17}$$

where χ and M are arbitrary integration constants.

The action (15) is invariant under local supersymmetry transformations (12) supplemented by transformation laws for $H_{\mu\nu\lambda}$ and $\Phi(B)$:

$$\delta_\epsilon H_{\mu\nu\lambda} = -e\, \varepsilon_{\mu\nu\lambda\kappa} \big(\bar{\varepsilon}\zeta^\kappa\big) \quad , \quad \delta_\epsilon \Phi(B) = \frac{\Phi(B)}{e} \delta_\epsilon e \ , \tag{18}$$

which algebraically close on-shell, *i.e.*, when Eq.(16) is imposed.

Supersymmetric Higgs Effect in Supergravity

The appearance of the integration constant M represents a ***dynamically generated cosmological constant*** in the pertinent gravitational equations of motion and, thus, it signifies a *spontaneous (dynamical) breaking of supersymmetry*.

Indeed, varying (15) w.r.t. e^a_μ:

$$e^{b\nu} R^a_{b\mu\nu} - \frac{1}{2}\bar{\psi}_\mu \gamma^{a\nu\lambda} D_\nu \psi_\lambda + \frac{1}{2}\bar{\psi}_\nu \gamma^{a\nu\lambda} D_\mu \psi_\lambda$$
$$+\frac{1}{2}\bar{\psi}_\lambda \gamma^{a\nu\lambda} D_\nu \psi_\mu + \frac{e^a_\mu}{2}\,\frac{\varepsilon^{\rho\nu\kappa\lambda}}{3!\,e}\,\partial_\rho H_{\nu\kappa\lambda} = 0 \qquad (19)$$

and using Eq.(17) (containing the arbitrary integration constant M) to replace the last H-term on the l.h.s. of (19), the results is:

Supersymmetric Higgs Effect in Supergravity

- We obtain the vierbein counterparts of the Einstein equations including a dynamically generated ***floating*** cosmological constant term $e^a_\mu M$:

$$e^{b\nu} R^a_{b\mu\nu} - \frac{1}{2} e^a_\mu R(\omega, e) + e^a_\mu M = \kappa^2 T^a_\mu \ ,$$

$$\kappa^2 T^a_\mu \equiv \frac{1}{2} \bar{\psi}_\mu \gamma^{a\nu\lambda} D_\nu \psi_\lambda - \frac{1}{2} e^a_\mu \bar{\psi}_\rho \gamma^{\rho\nu\lambda} D_\nu \psi_\lambda - \frac{1}{2} \bar{\psi}_\nu \gamma^{a\nu\lambda} D_\mu \psi_\lambda - \frac{1}{2} \bar{\psi}_\lambda \gamma^{a\nu\lambda} D_\nu$$

Recall: according to the classic paper *[Deser-Zumino, 78]* the sole presence of a cosmological constant in supergravity, even in the absence of manifest mass term for the gravitino, implies that the gravitino becomes ***massive***, i.e., it absorbs the Goldstone fermion of spontaneous supersymmetry breakdown – a ***supersymmetric Higgs effect***.

AdS Supergravity

- More interesting scenario: let us start with anti-de Sitter (AdS) supergravity:

$$S_{\mathrm{AdS-SG}} = \frac{1}{2\kappa^2}\int d^4x\, e\Big[R(\omega,e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda - m\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu - 2\Lambda_0\Big] \;,$$

$$m \equiv \frac{1}{L} \quad , \quad \Lambda_0 \equiv -\frac{3}{L^2} \;.$$

The action (21) contains additional explicit mass term for the gravitino as well as a bare cosmological constant Λ_0 balanced in a precise way $|\Lambda_0| = 3m^2$ so as to maintain local supersymmetry invariance and, in particular, keeping the ***physical gravitino mass zero***!

AdS Supergravity

Note: Here we have AdS space-time as a background with curvature radius L (unlike Minkowski background in the absence of a bare cosmological constant).
Therefore, the notions of “mass” and “spin” are given in terms of the Casimir eigenvalues of the UIR’s (discrete series) of the group of motion of AdS space $SO(2,3) \sim Sp(4,\mathbb{R})$ (for $D=4$) instead of the Poincare group ($SO(1,3) \ltimes R^4$) Casimirs.

Modified AdS Supergravity

- Now, let us apply the above TMT-formalism to construct a modified-measure AdS supergravity:

$$S_{\mathrm{mod-AdS-SG}} = \frac{1}{2\kappa^2}\int d^4x\,\Phi(B)\Big[R(\omega,e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda - m\,\bar{\psi}_\mu \gamma^{\mu\nu}\psi_\nu - 2\Lambda_0 + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda}\Big]\;, \qquad (23)$$

with $\Phi(B)$ as in (14) and m, Λ_0 as in (21). The action (23) is invariant under local supersymmetry transformations:

$$\delta_\epsilon e^a_\mu = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu \quad , \quad \delta_\epsilon\psi_\mu = \Big(D_\mu - \frac{1}{2L}\gamma_\mu\Big)\varepsilon\;,$$

$$\delta_\epsilon H_{\mu\nu\lambda} = -e\,\varepsilon_{\mu\nu\lambda\kappa}\big(\bar{\varepsilon}\zeta^\kappa\big) \quad , \quad \delta_\epsilon\Phi(B) = \frac{\Phi(B)}{e}\,\delta_\epsilon e\;. \qquad (24)$$

Modified AdS Supergravity - Principal Result

- The modified AdS supergravity action (23) will trigger dynamical spontaneous supersymmetry breaking resulting in the appearance of the dynamically generated floating cosmological constant M which will add to the bare Λ_0.
  Thus, we can achieve via appropriate choice of $M \simeq |\Lambda_0|$ a ***very small effective observable cosmological constant*** $\Lambda_{\rm eff} = M + \Lambda_0 = M - 3m^2 << |\Lambda_0|$ and, simultaneously, a ***large physical gravitino mass*** $m_{\rm eff}$ which will be very close to the gravitino mass parameter $m = \sqrt{|\Lambda_0|/3}$ since now background space-time geometry becomes almost flat.
  This is precisely what is required by modern cosmological scenarios for slowly expanding universe of today [A. Riess *et.al.*, S. Perlmutter *et.al.*].

Conclusions

- Two-measure formalism in gravity/matter theories (employing alternative non-Riemannian volume form, *i.e.* reparametrization covariant integration measure, on the spacetime manifold alongside standard Riemannian volume form) naturally generates a ***dynamical cosmological constant*** as an arbitrary dimensionful integration constant.
- Within modified-measure minimal $N = 1$ supergravity the dynamically generated cosmological constant implies spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect).

Conclusions

- Within modified-measure anti-de Sitter supergravity we can fine-tune the dynamically generated cosmological integration constant in order to achieve simultaneously a very small physical observable cosmological constant and a very large physical observable gravitino mass – a paradigm of modern cosmological scenarios for slowly expanding universe of today.

THANK YOU!

THANK YOU – Merci beaucoup

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