HIGHER SPIN GAUGE THEORIES, BACKGROUND INDEPENDENCE AND STRING FIELD THEORY

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- 2. Realization in String Theory and Relation to Hidden Global Symmetries
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1. GHOST COHOMOLOGIES: INTUITIVE DESCRIPTION

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Consider a gauge theory with gauge group G with the algebra g generated by $\{T^a\}; a = 1, ..., n$ satisfying

$$[T^a, T^b] = f_c^{ab} T^c$$

Consider the corresponding antighost field b^a of the opposite statistics in the adjoint of g and its canonical conjugate ghost field c^a, satisfying

$$\{b^a, c^b\} = \delta^{ab}$$
$$\{b^a, b^b\} = \{c^a, c^b\} = 0$$

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By definition, b^a carry ghost number -1 and c^a carry ghost number 1.

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Define the BRST charge Q of ghost number 1 according to:

$$Q = \sum_a c^a T_a - \frac{1}{2} \sum_{a,b,c} f_c^{ab} c_a c_b b^c \tag{1}$$

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It is straighforward to show that $Q^2 = 0$, provided that f_c^{ab} satisfy Jacobi identities.

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The gauge theory, symmetric under gauge transformations induced by T^a, is also symmetric under BRST transformations induced by Q

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The BRST symmetry transformations are structurally similar to the original gauge transformations, but have the opposite statistics, with the gauge parameters ϵ^a replaced by the ghost fields c^a.

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The physical states (local operators) $|\psi>$ of the gauge theory are the objects given by the elements of the BRST cohomology:

$$Q|\psi>=0$$
$$|\psi>\neq Q|\varphi>$$

where $|\varphi>$ is any other local operator constructed out of the fields of the theory.

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The ghost cohomology $H_n (n \neq 0)$ is defined as a subset of physical states (BRST cohomology) that carry ghost number n, such that:

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1.They are BRST-invariant but not manifestly gauge invariant, i.e. for any $|\psi>\subset H_n$

$$Q|\psi>=0$$
$$T^a|\psi>\neq 0 \tag{2}$$

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2. They are not related to any state of lower ghost number $n-1$ by any gauge and/or BRST transformation (for $n>0$) or to any state of higher ghost number $n+1$ (for $n<0$).

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3.H_0 is empty, i.e. any state of ghost number 0 the BRST-invariance always implies the gauge invariance.

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Ghost cohomologies are typically related to hidden global symmetries of the theory and/or contain crucial information about nonperturbative degrees of freedom

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For example, in QCD objects like those describe the gluon-ghost condensate, conjectured to play important role in quark confinement and emergence of mass gap in QCD (e.g. K.-I. Kondo, Phys.Lett. B572 (2003) 210-215)

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In string theory, ghost cohomologies are related to global symmetries, corresponding to hidden space-time dimensions, emergent AdS space, background independence and vertex operator realizations of higher-spin algebras in AdS spaces.

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In the rest of the talk, I shall limit myself to string-theoretic realization of the Ghost cohomologies.

2. Ghost Cohomologies - Definition and Realization in String Field Theory

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Consider RNS superstring theory (critical or noncritical) in d-dimensional flat space-time. The action in superconformal gauge is

$$\begin{aligned} S_{RNS} &= S_{matter} + S_{bc} + S_{\beta\gamma} + S_{Liouville} \\ S_{matter} &= -\frac{1}{4\pi}\int d^2z(\partial X_m \bar{\partial} X^m \\ &+\psi_m \bar{\partial}\psi^m + \bar{\psi}_m \partial \bar{\psi}^m) \\ S_{bc} &= \frac{1}{2\pi}\int d^2z(b\bar{\partial}c + \bar{b}\partial\bar{c}) \\ S_{\beta\gamma} &= \frac{1}{2\pi}\int d^2z(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma}) \\ S_{Liouville} &= -\frac{1}{4\pi}\int d^2z(\partial\varphi\bar{\partial}\varphi + \bar{\partial}\lambda\lambda \\ &+\partial\bar{\lambda}\bar{\lambda} + \mu_0 e^{B\varphi}(\lambda\bar{\lambda} + F)) \end{aligned}$$

where $X^m (m = 0, ...D-1)$ are the space-time coordinates; ψ^m are superpartners of X^m on the worldsheet, (b, c) are fermionic reparametrization ghosts and (β, γ) are superconformal ghosts. (φ, λ, F) are components of the super Liouville field and the Liouville background charge is $q = B + B^{-1} = \sqrt{\frac{9-D}{2}}$

- What are the global symmetries of this action?

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First of all, it is obviously invariant under Poincare algebra of translations and Lorentz rotations, generated by worldsheet integrals of conformal dimension 1 primary fields:

$$M^p = \oint \frac{dz}{2i\pi}\partial X^p(z)$$

$$M^{pq} = \oint \frac{dz}{2i\pi}(\frac{1}{2}X^{[p}\partial X^{q]} + \psi^p\psi^q) \tag{3}$$

•

Note that massless physical excitations in open and glosed string theory (such as a photon or a graviton) are related to the Poincare generators:

$$V_{ph}(k)$$

$$= A_p(k)\oint \frac{dz}{2i\pi}(\partial X^p + i(k\psi)\psi^p)e^{ikX}(z)$$

$$V_{gr}(k) = G_{pq}(k)\int d^2z(\partial X^p + i(k\psi)\psi^p)$$

$$\times(\bar\partial X^p + i(k\bar\psi)\bar\psi^q)e^{ikX}(z,\bar z)$$

so in the zero momentum limit $k = 0$ the photon is simplythe translation generator, while the graviton is bilinear in translations

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However, apart from the standard Poincare symmetries, the superstring action is also invariant under surprising ghost-matter mixing symmetries, realized non-linearly.

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Namely, bosonize the ghost fields according to:

$$
\begin{aligned}
c = e^{\sigma}(z); b = e^{-\sigma}(z) \\
\gamma = e^{\phi-\chi}(z); \beta = e^{\chi-\phi}\partial\chi(z)
\end{aligned}
\qquad (4)
$$

•

Here ϕ, χ and σ are free $2D$ bosons with the standard propagators:

$$
\begin{aligned}
&< \phi(z)\phi(w) >= - < \chi(z)\chi(w) > \\
&= - < \sigma(z)\sigma(w) >= -log(z-w)
\end{aligned}
$$

•

Then the RNS action is invariant under the following symmetry transformations: (D.P.,Phys.Rev. D82 (2010) 066005):

•

$$
\begin{aligned}
\delta X^p &= \alpha(\partial(e^\phi \psi^p) + 2e^\phi \partial \psi^p) \\
\delta \psi^p & \\
&= -\alpha(2\partial(e^\phi \partial X^p) + 2e^\phi \partial^2 X^p) \\
\delta \gamma &= \alpha e^{2\phi-\chi}(\psi_p \partial^2 X^p \\
&\quad -2\partial \psi_p \partial X^p) \\
\delta \beta &= \delta b = \delta c = 0
\end{aligned}
\tag{5}
$$

•

Under these transformations, the variation of the matter part of the superstring action is cancelled by that of the ghost part:

•

$$
\begin{aligned}
\delta S_{matter} &= -\delta S_{ghost} \\
= \int d^2 z [\bar{\partial}(e^{\phi})(\psi_p \partial^2 X^p \\
&-2\partial\psi_p \partial X^p)]
\end{aligned}
$$

•

The generator of these transformations is given by

•

$$
T = \oint \frac{dz}{2i\pi} e^{\phi}(\psi_p \partial^2 X^p - 2\partial\psi_p \partial X^p)
$$

•

There also exists a dual copy of the symmetry transformations obtained by replacing $\phi \to -3\phi$, with the generator given by

•

$$\tilde{T} = \oint \frac{dz}{2i\pi} e^{-3\phi}(\psi_p \partial^2 X^p - 2\partial\psi_p \partial X^p)$$

•

Remark 1: both T and $\tilde{T}$ are the integrals of dimension 1 primary fields, as conformal dimensions of both e^{ϕ} and $e^{-3\phi}$ are the same and equal to $-\frac{3}{2}$; the matter factor in the both of the generators is the same. It is a primary field of dimension $\frac{5}{2}$

•

Remark 2: Both T and $\tilde{T}$ commute with the Poincare generators (up to BRST-exact terms)

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Switching on the Liouville mode, it is straightforward to generalize the α-symmetry transformations to the vector generators. Namely, the RNS action is also invariant under the transformations generated by

•

$$T^m = \oint \frac{dz}{2i\pi} e^{\phi}(\lambda\partial^2 X^m - 2\partial\lambda\partial X^m + \partial^2\varphi\psi^m - 2\partial\varphi\partial\psi^m)$$

$$T^{mn} = \oint \frac{dz}{2i\pi}(\psi^m\psi^n - 2e^{\chi-\phi}\psi^{[m}\partial X^{n]} + 4ce^{2\chi-2\chi}\psi^m\psi^n)$$

•

As before, the ghost number -3 version of the vector T^m-generators is straightforward to construct, simply by replacing $\phi \to -3\phi$. The generators satisfy (D.P., Phys.Rev. D84 (2011) 126004)

•

$$[T^m, T^n] = -T^{mn} \tag{6}$$

•

The appearance of the minus sign is highly nontrivial, stemming from cumbersome calculations involving ghost picture-changing transformations The rest of the commutators are identical to those of the Poincare algebra.

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This means that altogether the T^m and T^{mn} generators realize the isometry algebra of the AdS space. It is important to stress that they all commute with the standard Poincare generators . This means that the RNS superstring action possesses both flat and AdS space global isometries, completely detached from each other

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An important property of the AdS isometry T^m-generators is their essential ghost coupling, as they carry either +1 or dual −3 $\beta - \gamma$ ghost charge. This ghost coupling cannot be removed by any gauge (picture-changing) transformation and is related to the appearance of the ghost cohomologies in string theory. Below I shall describe it in more details

VERTEX OPERATORS AND GHOST COHOMOLOGIES IN STRING THEORY

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The BRST operator in RNS string theory is straightforward to construct according to the original prescription and it is given by

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$$Q = \oint \frac{dz}{2i\pi}[cT - bc\partial c - \frac{1}{4}b\gamma^2$$
$$-\frac{1}{2}\gamma(\psi^p \partial X_p + \lambda\partial\varphi + \frac{q}{2}\partial\lambda)]$$

•

It is straightforward to check that $Q^2 = 0$. The physical states (vertex operators) $\{V\}$ in string theory, describing emissions of various particles and/or solitons by a string, are defined as the elements of the BRST cohomology:

•

$$\{Q, V\} = 0$$
$$V \neq \{Q, ...\} \tag{7}$$

•

Typically, the vertex operators in superstring theory are the objects of the form:

$$V(k) \sim \oint \frac{dz}{2i\pi}[e^{m\chi+n\phi+ikX}$$
$$P(\partial X, \partial^2 X, ..., \psi, \partial\psi, ...\partial\phi, \partial^2\phi, ...)$$

where k is the momentum, P is polynomial of a certain degree in derivatives of the matter and the bosonized ghost fields ϕ, χ and σ.

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The number $m + n$ (integer or half-integer) is called the **picture** of the vertex operator.

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In general, one and the same physical operator admits infinitely many different picture representations.

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That is, consider a physical vertex operator V_N at picture N. Given V_N, one can construct physically equivalent operators at higher or lower pictures by using direct and inverse picture-changing transformations. Namely,by using the direct and inverse picture-changing operators:

•

$$\begin{aligned} \Gamma =: \delta(\beta)G : (z) \\ = -\frac{1}{2}e^{\phi}\psi_m\partial X^m + \frac{1}{4}e^{2\phi-\chi}b(\partial\chi + \partial\sigma) \\ +ce^{\chi}\partial\chi(z) \\ \Gamma^{-1} = -4ce^{\chi-2\phi}\partial\chi(z) \\ :\Gamma^{-1}\Gamma := 1 \end{aligned} \quad (8)$$

•

one can raise and lower the ghost pictures using the normally ordered operator product expansions:

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$$V_{N+1} =: \Gamma V_N :$$
$$V_{N-1} =: \Gamma^{-1} V_N : \tag{9}$$

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Standard perturbative vertex operators in string theory (such as a photon) or Poincare generators can thus be represented at any positive or negative picture

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For example, the photon operators at pictures -2, -1 and 0 are

$$V_{-2} = A_p(k) \oint \frac{dz}{2i\pi} e^{-2\phi} \partial X^p e^{ikX}(z)$$
$$V_{-1} = A_p(k) \oint \frac{dz}{2i\pi} e^{-\phi} \psi^p e^{ikX}(z)$$
$$V_0(k) = A_p(k) \oint \frac{dz}{2i\pi} (\partial X^p + i(k\psi)\psi^p) e^{ikX}(z)$$

•

At $k = 0$ these expressions reduce to various picture representations of the translation generator in Poincare algebra. So for these operators the ghost dependence is merely an artefact of a gauge choice and can be removed by combination of picture transforms to picture zero.

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We will refer to these operators as elements of zero ghost cohomology H_0. In case of the AdS isometry generators , however, things are crucially different. Their ghost couplings are essential and cannot be removed by picture transforms, as they admit no zero picture representations.

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The AdS transvection T^m-generator exists at minimal picture $+1$ and can be transformed to any picture $N > 1$ by

$$T_N^m =: \Gamma^{N-1} T_1^m$$

, however it is annihilated by the inverse picture changing:

$$: \Gamma^{-1} T_1^m := 0$$

and does not admit representations at pictures below 1.

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Similarly, the picture -3 representation of the T^m-generator exists at minimal negative picture -3 and below; it can be transformed to any picture $N < -3$ by combination of $N-3$ inverse picture changing transformations:

$$T_N^m =: \Gamma^{3-N} T_{-3}^m :$$

however it is annihilated by direct picture changing transformation at picture -3:

$$: \Gamma T_{-3}^m := 0$$

and does not admit representations at pictures above -3

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A direct operator isomorphism, preserving BRST invariance, can be constructed between -3 and $+1$ representations of T^m (D.P. Phys.Rev. D82 (2010) 066005)

The AdS isometry transformations are convenient to classify in terms of **GHOST COHOMOLOGIES**, defined as follows:

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Positive ghost cohomologies H_n consist of physical (BRST-invariant and nontrivial) vertex operators existing at picture $n \geq 1$ and above, annihilated at minimal picture n by Γ^{-1}-transformation

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Negative ghost cohomologies H_{-n} consist of physical (BRST-invariant and nontrivial) vertex operators existing at picture $n \geq 3$ and below, annihilated at minimal negative picture$-n$ by Γ-transformation

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Positive and negative ghost cohomologies are isompophic (D.P. Phys.Rev. D82 (2010) 066005) :

$$H_n \sim H_{-n-2} (n \geq 1)$$

•

H_{-1} and H_{-2} are empty while H_0 consists of picture-equivalent operators, admitting picture 0 representation.

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The Poincare symmetry algebra in superstring theory is thus generated by the operators of H_0 while AdS isometry algebra is generated by operators of $H_1 \sim H_{-3}$.

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Given the hidden global space-time symmetries of superstring theory, stemming from $H_1 \sim H_{-3}$-generators, are there also the global symmetries associated with operators of higher rank cohomologies $H_n \sim H_{-n-2}$ with $n \geq 2$?

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The answer to this question is positive: In particular, it is straightforward to show that the RNS action is invariant under global transformations generated by the following currents of $H_{s-2} \sim H_{-s}(s \geq 3)$:

$$
\begin{aligned}
T^{s-1|s-2} &\equiv T^{a_1 \dots a_{s-1}|b_1 \dots b_{s-2}} \\
= \oint \frac{dz}{2i\pi} e^{(s-2)\phi} \partial\psi_{(b_1} \partial^2 \psi_{b_2} \dots \partial^{s-3} \psi_{b_{s-2})} \\
\times \partial X_{a_1} \dots \partial X_{s-1} \\
\sim \oint \frac{dz}{2i\pi} e^{-s\phi} \partial\psi_{(b_1} \partial^2 \psi_{b_2} \dots \partial^{s-3} \psi_{b_{s-2})} \\
\times \partial X_{a_1} \dots \partial X_{s-1}
\end{aligned}
$$

•

These currents are the two-row operators, symmetric in a and b-indices. Combined with the AdS isometry generators of $H_1 \sim H_{-3}$, the global symmetries induced by $H_{s-2} \sim H_{-s}(3 \leq s \leq \infty)$ generators form the infinite-dimensional algebra.

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The AdS isometry algebra is the maximal finite-di mensional subalgebra of this algebra. The full symmetry algebra of the larger superstring theory turns out to be isomorphic to the **higher spin algebra in** AdS

HIGHER SPIN FIELDS AND VASILIEV'S FRAME-LIKE FORMALISM

In the simplest formulation, the higher spin field theory is the theory of symmetric tensor fields of rank s satisfying the Pauli-Fierz on-shell conditions:

$$\begin{aligned} \partial_p \partial^p H^{m_1 \dots m_s}(x) &= 0 \\ \partial_{m_1} H^{m_1 \dots m_s}(x) &= 0 \\ H^m_m m_1 \dots m_{s-2} &= 0 \end{aligned} \tag{10}$$

invariant under the gauge transformations with traceless and divergence free rank $s-1$ gauge parameter:

$$\delta H^{m_1 \dots m_s} = \partial^{(m_s} \Lambda^{m_1 \dots . m_{s-1})} \tag{11}$$

Because of the vast gauge symmetry, necessary to eliminate negative-norm states, constructing consistent gauge-invariant interacting HS theories is a highly nontrivial and complicated problem, to large extent unresolved (with the exception of few examples) Even in the non-interacting case the action was not known until after 1978 when it was constructed by Fronsdal:

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$$
\begin{aligned}
S = \int d^D x \{ & \partial_m H_{m_1 \ldots m_s} \partial^m H^{m_1 \ldots m_s} \\
& -(s-1) s \partial_m H^n_{n m_3 \ldots m_s} \partial^m H_n^{n m_3 \ldots m_s} \\
& +s(s-1) \partial_m H^n_{n m_3 \ldots m_s} \partial_p H^{p m m_3 \ldots m_s} \\
& -s \partial_m H_{m m_2 \ldots m_s} \partial_n H^{n m_2 \ldots m_s} \\
& -\frac{(s-2)(s-1)s}{4} \partial_m H_n^{n m m_4 \ldots m_s} \partial_p H_q^{p q m_4 \ldots m_s} \}
\end{aligned}
$$

•

In the interacting case, Coleman-Mandula's theorem forbids the existence of consistent interacting theories beyond spin 2, unless one compromises on locality and unitarity or considers these theories in the AdS space where no well-defined boundary S-matrix exists, making it possible to circumvent the assumptions of the theorem.

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Higher spin fields in AdS are known to constitute crucial a ingredient of AdS/CFT correspondence and holography principle , as they are related to the major part of the operators on the CFT side.

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Higher spin fields also inevitably appear in theories involving extra space-time dimensions. For this reason understanding their dynamics is a deep and profound physical problem.

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It turns out that the higher spin interacting theories are far more natural objecta to describe using Vasiliev's frame-like formalism (a higher-spin extension of Cartan-Weyl frame description of gravity using vielbeins and spin connections instead of the metric) then the metric approach (in particular, used by Fronsdal).

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String theory turns out to be a particularly efficient and promising framework to describe higher spin dynamics; the vertex operator description of higher spin modes in superstring theory naturally fits the frame-like formalism

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In the frame-like formalism, a symmetric higher spin gauge field of spin s is described by collection of two-row fields $\Omega^{s-1|t} \equiv \Omega_m^{a_1 \ldots a_{s-1}|b_1 \ldots b_t}(x)$ with$0 \leq t \leq s-1$ and the rows of lengths $s-1$ and t.

•

The only truly dynamical field of those is $\Omega^{s-1|0}$ while the fields with $t \neq 0$, called the extra fields, are related to the dynamical one through **generalized zero torsion constraints**:

$$\Omega^{s-1|t} \sim D^{(t)}\Omega^{s-1|0} \tag{12}$$

where $D^{(t)}$ is the order t linear differential operator preserving the symmetries of the appropriate Yang tableaux. There are altogether $s-1$ constraints for the field of spin s.

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As for the dynamical
$\Omega^{s-1|0}$-field (symmetric in all the a-indices), it splits into two diagrams with respect to the manifold m-index.

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Assuming the appropriate pullbacks, the one-row symmetric diagram describes the dynamics of the $metric-like$ symmetric Fronsdal's field of spin s while the two-row component of $\Omega^{s-1|0}$ can be removed by appropriate gauge transformation.

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The gauge transformations and transversality constraints on Ω are given by

$$\delta\Omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t} = D_m\epsilon^{a_1\ldots a_{s-1}|b_1\ldots b_t}$$
$$\eta_{a_1a_2}\Omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t} = 0$$

•

The generalized Cartan's 1-form is defined according to

$$U = (e_m^a(x)T_a + \omega_m^{ab}T_{ab}$$
$$+\sum_{s=3}^{\infty}\sum_{t=0}^{s-1}\Omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t}T_{a_1\ldots a_{s-1}|b_1\ldots b_t})dx^m$$

•

Here e_m^a and ω_m^{ab} are vielbein and spin 2 connection, Ω's are frame-like higher spin fields. T_a and T_{ab} are isometry generators of underlying space-time (such as AdS_D).

•

Combined with these isometry generators, the higher spin currents $T_{a_1 \ldots a_{s-1}|b_1 \ldots b_t}$ generate **infinite-dimensional higher spin algebra** on AdS_D

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Some examples: for $D = 2$ the HS algebra is isomorphic to Virasoro algebra, for $D = 3$ it is related to w_∞; for higher D's the higher spin algebras are more complicated objects.

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Structurally, the HS algebras are given by

$$[T^{s_1|t_1}, T^{s_2|t_2}] \sim \sum_{s=|s_1-s_2|+1}^{s_1+s_2-2} \sum_{t=0}^{s-1} C^{s_1,s_2|t_1,t_2}_{s|t} T^{s|t} \tag{13}$$

The generalized higer spin curvature 2-forms are defined according to

$$R_{mn}^{a_1 \ldots a_{s-1}|b_1 \ldots b_t} \equiv R^{s-1|t} = d\Omega^{s-1|t} + \sum_{s_1,s_2,t_1,t_2} (\Omega^{s_1-1|t_1} \wedge \star \Omega^{s_2-1|t_2})^{s|t}$$

Here $\star$ is the associative product in the higher spin algebras. Note that a curvature of any rank is contributed by infinite number of terms, originating from the 1-forms of any spin value.

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The higher spin action is bilinear in R. Its complete form is unknown although many approximations exist, particularly, in the linearized limit.

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String theory provides a remarkable realization of higher spin algebras in AdS in terms of vertex operators from ghost cohomologies $H_{s-2} \sim H_{-s}$. These vertex operators live in the "larger" string theory and are based on infinite dimensional matter-ghost mixing global symmetries of string theory, induced by the generators described above.

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Higher Spin algebra in AdS_D is realized as the operator algebra of vertex operators for the frame-like higher spin gauge fields in noncritical D-dimensional RNS superstring theory.

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The OPE fusion rules for the vertex operators of nonzero ghost cohomologies are identical to the structure of the higher spin algebras:

$$[H_{s_1}](z_1) \otimes [H_{s_2}](z_2) \sim$$
$$(z_1 - z_2)^{-1} \sum_{s=|s_1-s_2|+1}^{s_1+s_2-2} [H_s](\frac{z_1+z_2}{2})$$
$$+[Q_{brst}, ...]$$

•

Explicit Construction of the VERTEX OPERATORS of higher cohomologies (D.P. Phys. Rev. D. 89, 026010 (2014)):

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$\mathrm{H}_{-3} \sim H_1$

$$
\begin{aligned}
V_{ph}^{AdS} = A_m(p) \oint \frac{dz}{2i\pi} e^{-3\phi} R^m e^{ipX} \\
\equiv \oint \frac{dz}{2i\pi} e^{-3\phi+ipX} \\
\times\{\lambda\partial^2 X^m - 2\partial\lambda\partial X^m \\
+\partial^2\varphi\psi^m - 2\partial\varphi\partial\psi^m \\
+ip^m(\frac{1}{2}\partial^2\lambda + \frac{1}{q}\partial\varphi\partial\lambda \\
-\frac{1}{2}\lambda(\partial\varphi)^2 + \\
(1+3q^2)\lambda(3\partial\psi_p\psi^p - \frac{1}{2q}\partial^2\varphi))\}
\end{aligned}
$$

•

This massless open string operator is the element of $H_1 \sim H_{-3}$ describes the emission of a photon by an open string in AdS background, polarized along the AdS boundary, and must not be confused with usual photon excitation in flat string theory (the element of H_0)

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Just as the usual photon reduces to translation operator in flat space at $p = 0$, the massless $s = 1$ operator in our model is reduced to AdS transvection generator at zero momentum.

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Next, in close string theory one is able to construct the the spin 2 (graviton) vertex operator at ghost cohomology $H_1 \otimes \bar{H}_1 \sim H_{-3} \otimes \bar{H}_{-3}$ describing the gravitational fluctuations around the AdS_D vacuum, polarized along the AdS boundary (not to be confused with the "standard" string theory graviton, that is the element of $H_0 \otimes \bar{H}_0$) and describes fluctuations around the flat vacuum. Just as the standard graviton is the object bilinear in flat space translations, the $H_1 \otimes \bar{H}_1$ graviton in our case is bilinear in AdS transvections.

•

The explicit expression for this operator is

$$V_{grav}^{AdS}(p)$$
$$= G_{mn}(p) \int d^2 z e^{-3\phi - 3\bar{\phi} + ipX} R^m \bar{R}^n$$

•

Next, the vertex operators for the frame-like symmetric higher spin s fields in AdS space are the elements of $H_{s-2} \sim H_{-s}$ in open string theory and are given by

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$$V_s = \Omega_{a_1 \ldots a_{s-1}|b_1 \ldots b_{s-2}}(p) \oint \frac{dz}{2i\pi} e^{-s\phi + ipX} \partial X^{a_1} \ldots \partial X^{a_{s-1}} \times \psi^{b_1} \partial \psi^{(b_2} \ldots \partial^{(s-3)} \psi^{b_{s-3})}(z)$$

These are the operators for the $\Omega^{s-1|s-2}$ extra field in the frame-like formalism.

•

The operators for the remaining extra fields $\Omega^{s-1|t}$ $(0 \leq t \leq s-3)$ are related to those for $\Omega^{s|s-3}$ through generalized zero curvature relations in terms of ghost cohomology conditions:

$$\Omega_{s-1|s-2}(p) V^{s-1|s-2} = \Omega_{s-1|t}(p) : \Gamma^{s-2-t} V^{s-1|t} : \tag{14}$$

•

Note that the operators $V^{s-1|t}$ for $\Omega^{s-1|t}$ have negative canonical ghost pictures $t+2-2s$, however they are the elements of H_{-s}. This cohomology condition entails the zero torsion conditions for all the Ω's in the list. The manifest expressions for $t < s-2$ are complicated.

•

BRST invariance conditions $[Q, V] = 0$ for the above vertex operators lead to onshell equations of motion for A_m, G_{mn} and $\Omega^{s-1|t}$ in AdS space.

•

The BRST nontriviality conditions V$\neq\{Q, U\}$ lead to the gauge transformations on Ω's: each gauge transformation on Ω leads to shifting the corresponding vertex operators by BRST-exact terms irrelevant for the correlation functions.

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Therefore the correlation functions of the vertex operators for the higher spin fields, computed in string theory, produce the higher spin interaction terms, gauge-invariant by construction.

CONTRIBUTIONS to the β-FUNCTION.

In the leading order, the linearized contributions follow from the Weyl invariance constraints on the vertex operators. The calculations are the standard ones,e.g., using the ϵ-expansion techniques. However, the crucial novelty in our case is the appearance of the cosmological term. The cosmological term for β_{mn} appears as a result of nontrivial ghost dependence of V^{mn}, i.e. as a result of V^{mn} being an element of nonzero cohomology $H_1 \otimes \bar{H}_1$. Namely, the Weyl invariance constraints can be conveniently deduced from the OPE:

$$\sim \int d^2 z \int d^2 w T_{z\bar{z}}(z,\bar{z}) V_{grav}(w,\bar{w})$$

by expanding around the midpoint and evaluating the coefficient in front of

$$\sim \frac{V_{grav}(\frac{z+w}{2}, \frac{\bar{z}+\bar{w}}{2})}{|z-w|^2}$$

(note that the trace $T_{z\bar{z}}$ of the stress-energy tensor, generating the Weyl transformation, is nonzero in the underlying ϵ-expansion). For a usual graviton operator

$$\sim G_{mn}(p) \int d^2 w \partial X^m \bar{\partial} X^n e^{ipX}(w,\bar{w})$$

in the bosonic string this procedure leads, after simple calculation, to the standard β-function contribution, quadratic in momentum, given by the linearized part of the Ricci tensor plus the second derivative of the dilaton $\sim R^{lin}_{mn} - 2p_m p_n D$ with the dilaton $D \sim tr(G_{mn})$. The calculation, leading to the identical result, is similar in superstring theory. The graviton operator must then be taken at canonical ghost picture (unintegrated $b-c$ picture and $(-1,-1)$ $\beta-\gamma$ ghost picture), so

$$V_{grav} = c\bar{c}e^{-\phi-\bar{\phi}}\psi^m\psi^n e^{ipX}$$

and

$$\begin{aligned} T_{z\bar{z}} \equiv T^{matter}_{z\bar{z}} \\ +T^{b-c}_{z\bar{z}} + T^{\beta-\gamma}_{z\bar{z}} \\ = -\frac{1}{2}(\partial X_m \bar{\partial} X^m - \bar{\partial}\psi_m \psi^m \\ -\partial\bar{\psi}_m\bar{\psi}^m + \partial\sigma\bar{\partial}\sigma \\ +\partial\chi\bar{\partial}\chi - \partial\phi\bar{\partial}\phi) \end{aligned}$$

The OPE of V_{grav} with $T_{z\bar{z}}^{matter}$ then contributes the term $\sim p^2 G_{mn}$ to the graviton's beta-function (which is the gauge-fixed linearized part of the Ricci tensor, with the gauge condition $\sim p^m G_{mn} = 0$), while the contribution stemming from the OPE with $T_{z\bar{z}}^{b-c}$ cancels the one from the OPE with $T_{z\bar{z}}^{\beta-\gamma}$ since

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$$\begin{aligned}
&\partial\sigma\bar{\partial}\sigma(z,\bar{z})c\bar{c}(w,\bar{w}) \\
&\sim \frac{1}{|z-w|^2}c\bar{c}(w,\bar{w}) \\
&\partial\phi\bar{\partial}\phi(z,\bar{z})e^{-\phi-\bar{\phi}}(w,\bar{w}) \\
&\sim \frac{1}{|z-w|^2}e^{-\phi-\bar{\phi}}(w,\bar{w})
\end{aligned}$$

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and σ and ϕ-terms of $T_{z\bar{z}}$ have opposite signs.

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It is this cancellation that ensures the absence of "cosmological terms" in the β-function of the graviton with the conventional vertex operator leading to Einstein gravity around the flat vacuum. In case of the vertex operator for the AdS graviton, the OPE of $T_{z\bar{z}}^{matter}$ with $V_{grav}^{H_{-3}\otimes H_{-3}}$ still leads to the linear part of the Ricci tensor.However, since this operator is the element of $H_{-3}\otimes H_{-3}$, and its canonical ϕ-ghost picture is $(-3,-3)$ (refs), the contributions from $T_{z\bar{z}}^{b-c}$ and $T_{z\bar{z}}^{\beta-\gamma}$ no longer cancel each other:

- $$(T_{z\bar{z}}^{b-c}+T_{z\bar{z}}^{\beta-\gamma})(z,\bar{z})V_{grav}^{H_{-3}\otimes H_{-3}}(w,\bar{w}) \sim \frac{\frac{1}{2}(1-3^2)V_{grav}^{H_{-3}\otimes H_{-3}}}{|z-w|^2}$$

  leading to the cosmological term proportional to $\sim 4G_{mn}$ in the β-function. Thus the Weyl invariance condition brings the piece proportional to

  $$\sim R_{mn}^{linearized}+4g_{mn}$$

  to the β-function (assuming that the dilaton is switched off).

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The computation of the leading order β-function contribution from the higher spin vertex operators is analogous, leading to the following result for the Fronsdal metric-like field:

$$\beta_m^{a_1 \ldots a_{s-1}} \equiv \Lambda \frac{d}{d\Lambda} \Omega_m^{a_1 \ldots a_s} = -p^2 \Omega_m^{a_1 \ldots a_{s-1}}(p)$$
$$+\Sigma_1(a_1|a_2, \ldots a_{s-1}) p_t p^{a_1} \Omega_m^{a_2 \ldots a_{s-1} t}$$
$$-\frac{1}{2} \Sigma_2(a_{s-2}, a_{s-1}|a_1, \ldots, a_{s-3}) p^{a_{s-1}} p^{a_{s-2}}$$
$$\times (\Omega'_m)^{a_1 \ldots a_{s-3}} - 4(s-1) \Omega_m^{a_1 \ldots a_{s-1}}$$

where $\Sigma_{1,2}$ are the Fronsdal's symmetrization operators.

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This precisely gives the equations of motion for the Fronsdal's higher spin action in AdS space, for the higher spin fields polarized along the AdS boundary. Again, the ghost cohomology structure of the higher spin vertex operator is crucial to ensure the emergent AdS geometry (D.P. Phys. Rev. D. 89, 026010 (2014))

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The higher order contributions to the β-function are given by the worldsheet N-point correlators of the higher spin vertex operators. At this point we mostly have been able to compute the cubic couplings, although some particular quartics for the higher spin were computed as well (D.P., Phys.Rev. D 2011)

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At the cubic order, in AdS_4, the structure constants of HS algebra, computed this way, reproduce the holographic cubic couplings of $O(N)$ vector model in $d = 3$. In AdS_5, the structure constants particularly reproduce the gradient expansion in holographic hydrodynamics in $d = 4$

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Calculations beyond the cubic order generally require the modified off-shell string field theory techniques , extended to higher ghost cohomologies (D.P., in preparation)

5. Conclusions and outlook

- RNS superstring theory in is invariant under infinite-dimensional algebra of global symmetries, identified with higher spin algebra in AdS_D. This is despite the fact that the original theory is defined in Minkowski space. This is the first known physical realization of unbroken higher spin symmetries (D.P. Phys. Rev. D. 89, 026010 (2014); also to appear)

- The higher spin algebra is realized as a vertex operator algebra in superstring theory. The vertex operators describe emissions of frame-like higher spin gauge fields in AdS space by a string

- The Weyl invariance conditions on the vertex operators, defining the leading order contribution to the conformal β-function, lead to the appearance of the cosmological constant in the gravity part and to the Fronsdal's EOM for the higher spins in AdS. The β-function calculations reveal the emergent AdS geometry.

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The structure constants of higher spin algebra in AdS are identified with the three-point correlation functions $< V_{s_1} V_{s_2} V_{s_3} >$ in $2D$ CFT of RNS string theory. These correlation functions are tedious but straightforward to compute. Given the BRST constraints on V_S, these correlators reproduce higher-spin interactions by constructions.

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In AdS_4, the structure constants of HS algebra, computed this way, reproduce the holographic cubic couplings of $O(N)$ vector model in $d = 3$. In AdS_5, the structure constants particularly reproduce the gradient expansion in holographic hydrodynamics in $d = 4$

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The appearance of the ghost cohomologies H_s which OPE fusion is identical to HS algebra structure, as well as the emergence of the AdS space is an important hint towards background-independence of the larger string theory. One can hope to find new symmetries and vertex operators corresponding to emergent backgrounds other than AdS and to formulate consistent HS theories in these geometries as well

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The off-shell string field theory extended to nonzero ghost cohomology is the crucial object to explore in this direction. The key ingredient is the analytic solutions of SFT e.o.m.: $Q\psi - \psi \star \psi$ in higher ghost cohomologies

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The solutions relevant to emergent AdS backgrounds are generally of the form (D.P., to appear):

$$\Psi \sim \sum_{N,n} \lambda_{nN} c e^{-N\phi} B^{(n)}(\sigma, \chi, \phi)$$

where B are the Bell polynomials (so far, better known in cryptography rather than in CFT) and λ are coefficients given by recurrence relations, similar to Hardy-Ramanujan formula for partitions

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Different types of analytic SFT solutions may hold the key to strings and higher spin gauge fields in various geometries, leading to new types of holographic relations