Particle creation multiplicity in modified AdS_5 spaces

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base on I. Ya. Aref'eva, E. O. Pozdeeva, T. O. Pozdeeva, arXiv:1401.1180 [hep-th].

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QGP

Quark Gluon Plasma (QGP) has been discovered in Au+Au collision at energy 100 GeV for nucleon in 2005 @ RHIC

QGP formation

- In Early Universe
- In Heavy Ions Collisions

5D gravity and 4D field theory are related

- In holographic approach classical gravity in describes strong coupling field theory in 4D Minkowski space.
- There is hypothesis that QGP formation in 4D space corresponds to Black Holes creation in dual 5D space.

Maldacena, 9711200
Gubser, Klebanov, Polyakov, 9802109
Witten, 9802150

The gravitational shock wave in AdS_5 space is dual to ultrarelativistic heavy-ion in 4D space-time.
Thus,

- heavy-ion collisions can be represented such as gravitational shock waves collisions in AdS_5
- QGP formation is equivalent BH creation in AdS_5

Gubser et all
0805.1551; 0902.4026

Multiplicity and trapped surface area

- Main conjecture: multiplicity is proportional to entropy
  Gubser et all 0805.1551

- On experiments can be measured only $N \sim N_{ch}$
  B.B. Back et al., 0210015[nucl-ex]

- According experiment at $Pb-Pb$ and $Au-Au$ collisions the multiplicity is proportional to nucleon collision energy in 0.3 power: $N_{ch} \sim s_{NN}^{0.15}$, $\sqrt{s_{NN}} = 2E$, E is energy of collision nucleons (the experimental data was considered at energy $10-10^3$ GeV)

  K. Aamodt et al.[ALICE Collaboration], 1011.3916 [nucl-ex]

- However, the simplest holographic model gives another result $N_{ch} \sim s_{NN}^{1/3} \sim s_{NN}^{2/3}$

  Gubser et all 0805.1551

- The minimal black hole entropy can be estimated by trapped surface area

$$S \geq S_{trapped} = \frac{A_{trapped}}{4G_N}$$

- The trapped surface is surface whose null normals all propagate inward.

S. W. Hawking and D. Page, Thermodynamics Of Black Holes In Anti-de Sitter Space, Commun. Math. Phys. 87 (1983) 577.
C. S. Peëca, J. P. S. Lemos, 9805004 [gr-qc]

- Modification of holographic AdS_5 space by the introduction of b-factor to the initial metric including shock wave metric can gives another estimation for the multiplicity

$$S_{trapped} = \frac{\int \sqrt{det|g_{AdS_3}|} dz dx_\perp}{2G_5}$$

Kiritsis, 1111.1931

- The collisions of shock waves with masses averaged over transversal surfaces can be named domain-wall or domain collisions.

Shuryak et al., 1011.1918 [hep-th].
Arefeva et al., 1201.6542 [hep-th].

- In the AdS_5 model the scalar fields and corresponding potentials are absent.
- However the scalar field and potential can exist for the modified AdS_5 spaces with b-factor.
- We consider the action of five-dimensional gravity coupled to a scalar dilaton field in the presence of a negative cosmological constant

$$S_5 = S_R + S_\Phi ,$$

S_R is the Einstein-Hilbert action with the negative cosmological constant

$$S_R = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[R + \frac{d(d-1)}{L^2}\right] dx^5 ,$$

$d+1 = D = 5$, S_ϕ is the dilaton action,

$$S_\Phi = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[-\frac{4}{3}(\partial\Phi)^2 + V(\Phi)\right] dx^5 .$$

In the assumption that the background metric has the form

$$ds^2 = b^2(z)\big(dz^2 + dx^i\, dx^i - dx^+\, dx^-\big), \qquad i = 1, 2,$$

the Einstein equations reduce to two independent relations between field and potential with b-factor:

$$\Phi' = \pm\frac{3}{2}\sqrt{\left(\frac{2(b')^2}{b^2} - \frac{b''}{b}\right)},$$

$$V(\Phi(z)) = \frac{3}{b^2}\left(\frac{b''}{b} + \frac{2(b')^2}{b^2} - \frac{4b^2}{L^2}\right).$$

Shock wave

- To deal with a point-like shock wave

$$ds^2 = b^2(z)\big(dz^2 + dx^i\, dx^i - dx^+\, dx^- + \phi(z, x^1, x^2)\delta(x^+)(dx^+)^2\big), \ i = 1, 2,$$

((x^+, x^-, x^i, z) are light-like coordinates)

- the action of a point-like source moving along a trajectory $x^\mu = x_*^\mu(\eta)$ should be added to the initial action

$$S_{\rm st} = \int \left[\frac{1}{2e} g_{\mu\nu} \frac{dx_*^\mu}{d\eta} \frac{dx_*^\nu}{d\eta} - \frac{e}{2} m^2 \right] d\eta,$$

where m is the particle mass, η is an arbitrary world-line parameter, the particle mass is ussumed zero, which allows treating only with light-like geodesics,
e_μ^a is the frame associated with the metric, $g_{\mu\nu} = e_\mu^a e_{\nu\, a}$, and e is the square root of its determinant $e = \sqrt{-g}$.

S.S. Gubser et. all arXiv:0902.4062 [hep-th].

- The shock wave profile $\phi(z, x_\perp)$ solves the additional Einstein equation

$$\left(\partial^2_{x^1} + \partial^2_{x^2} + \partial^2_z + \frac{3b'}{b} \partial_z \right) \phi(z, x_\perp) = -16\pi G_5 \frac{E}{b^3}\, \delta(x^1)\delta(x^2)\delta(z - z_*).$$

The domain

- The assumption about the domain is a disk with masses averaged over transversal surface of radius L allows to transform the shock wave profile equation into the form

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^\omega(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z - z_*), \quad \text{where } E^* = \frac{E}{L^2}.$$

- The conditions on the boundary points z_a, z_b of the trapped surface formation is generalized to modified AdS_5 space

$$(\partial_z\phi^\omega)\big|_{z=z_a} = 2, \qquad (\partial_z\phi^\omega)\big|_{z=z_b} = -2,$$

where $z_a < z_* < z_b$, z_* is collision point.

The solution of domain profile equation is given as

$$\phi^{\omega}(z) = \phi_a \Theta(z_* - z) + \phi_b \Theta(z - z_*),$$

where

$$\phi_a = \frac{16\pi G_5 E}{L^2} \cdot \frac{\int_{z_b}^{z_*} b^{-3} dz \cdot \int_{z_a}^{z} b^{-3} dz}{\int_{z_b}^{z_a} b^{-3} dz}, \quad \phi_b = \frac{16\pi G_5 E}{L^2} \cdot \frac{\int_{z_a}^{z_*} b^{-3} dz \cdot \int_{z_b}^{z} b^{-3} dz}{\int_{z_b}^{z_a} b^{-3} dz}.$$

Using the conditions of the trapped surface formation, we get

$$\frac{8\pi G_5 E}{L^2} b^{-3}(z_a) \frac{\int_{z_b}^{z_*} b^{-3} dz}{\int_{z_b}^{z_a} b^{-3} dz} = 1, \quad \frac{8\pi G_5 E}{L^2} b^{-3}(z_b) \frac{\int_{z_a}^{z_*} b^{-3} dz}{\int_{z_b}^{z_a} b^{-3} dz} = -1.$$

Using the designation $\int_{z_i}^{z_j} b^{-3} dz = F(z_j) - F(z_i)$ we obtain the relations between points z_*, z_a, z_b and z_a, z_b accordingly

$$F(z_*) = \frac{b^{-3}(z_b) F(z_a) + b^{-3}(z_a) F(z_b)}{b^{-3}(z_a) + b^{-3}(z_b)}, \quad b^{-3}(z_a) = \frac{b^{-3}(z_b)}{\frac{8\pi G_5 E}{L^2} b^{-3}(z_b) - 1}.$$

In the follows, we calculate the relative entropy s

$$s = \frac{S_{\text{trap}}}{\int d^2 x_{\perp}} = \frac{1}{2G_5} \int_{z_a}^{z_b} b^3 \, dz.$$

fixing the point z_b, calculating z_a

Power-law b-factor, relative entropy

The relative entropy for power-law b-factor $b(z) = (L/z)^a$

$$s = \frac{1}{2G_5(3a-1)}\left(z_a\left(\frac{L}{z_a}\right)^{3a} - z_b\left(\frac{L}{z_b}\right)^{3a}\right).$$

The boundary trapped surface point z_a can not be fixed but found from the system for z_*, z_a with a given z_b:

$$z_a = \left(\frac{z_b^{3a}}{-1+z_b^{3a}C}\right)^{1/3a}, \quad z_* = \left(\frac{z_a^{3a}z_b^{3a}(z_b+z_a)}{z_a^{3a}+z_b^{3a}}\right)^{1/(3a+1)}, \quad C = \frac{8\pi G_5 E}{L^{3a+2}}$$

We consider $z_a \ll z_* \ll z_b$ and have the approximation:

$$z_a \sim \left(\frac{1}{C}\right)^{1/3a}, \qquad z_* \sim \left(\frac{z_b}{C}\right)^{1/(3a+1)}.$$

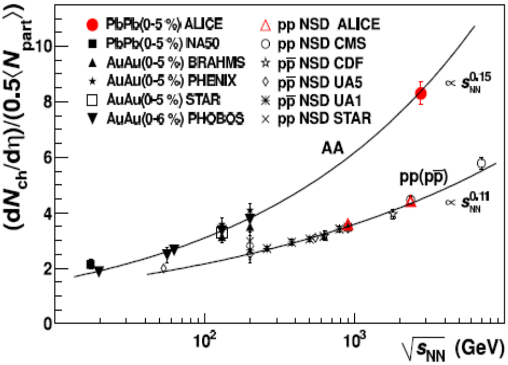
With the assumption $3a > 1$ the relative entropy tends to its maximum value at infinite z_b:

$$s|_{z_b\to\infty} = \frac{L^{3a}}{2G_5(3a-1)}z_a^{1-3a} = \frac{L}{2(3a-1)G_5}\left(\frac{8\pi G_5}{L^2}\right)^{(3a-1)/3a} E^{(3a-1)/3a}.$$

We thus find that for $a > 1/3$, the entropy S increases as $E^{(3a-1)/3a}$.

Power-law b-factor, comparing with experiments

- The multiplicity of particles produced in collisions of heavy ions (PbPb-and AuAu-collisions) dependents on energy as $(N \sim E^{0.3} \sim s_{NN}^{0.15})$ in the range $10 - 10^3$ GeV.
  K. Aamodt et al. [ALICE Collaboration], 1011.3916 [nucl-ex].



- The model with power-law wrapping factor can coincide with experimental data at $a \approx 0.47$

Power-law b-factor, potential

- For power-law b-factor $b(z)=(L/z)^a$ the potential and fields can be represented explicitly thought variable z. Since in this case $\Phi=\Phi(z)$ is single-valued function we can find $z=z(\Phi)$ and substitute it to the expression for potential $V(z)$ to get

$$V(\Phi)=-\frac{12}{L^2}+\frac{3a(3a+1)}{L^{2a}}\exp\left(\pm\frac{4}{3}\sqrt{\frac{(a-1)}{a}}(\Phi-\Phi_0)\right).$$

 1. $V(\Phi)$ is real for $a>1$.
 2. If $a=1$ we have AdS_5 space.
 3. If $a<1$ we consider the phantom field Φ_p with the action

$$S_{\Phi_p}=-\frac{1}{16\pi G_5}\int\sqrt{-g}\left[\frac{4}{3}(\partial\Phi_p)^2+\tilde{V}(\Phi_p)\right]dx^5.$$

The phantom field is related with the dilaton field Φ via $\Phi-\Phi_0=i(\Phi_p-\Phi_{p_0})$, and the potential for $a<1$ becomes

$$\tilde{V}(\Phi_p)=-\frac{12}{L^2}+\frac{3a(3a+1)}{L^{2a}}\exp\left(\pm\frac{4}{3}\sqrt{\frac{1-a}{a}}(\Phi_p-\Phi_{p_0})\right).$$

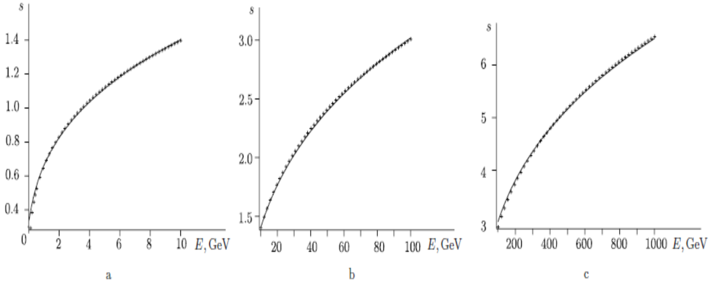
Mixed b-factor $b(z)=\left(\frac{L}{z}\right)^{a}\exp\left(-z^2/R^2\right)$, relative entropy

- In considering case the relative trapped surface area is $s=$
$$\left.\frac{\left(\frac{L}{z}\right)^{3a} z\exp\left(-\frac{3z^2}{2R^2}\right)\left(2\left(\frac{3z^2}{R^2}\right)^{\frac{3a-1}{4}}\mathbf{M}\left(\frac{-3a+1}{4},\frac{3(-a+1)}{4},\frac{3z^2}{R^2}\right)+3(1-a)\exp\left(-\frac{3z^2}{2R^2}\right)\right)}{2G_5\cdot 3(3\,a-1)(a-1)}\right|_{z_a}^{z_b},$$
where $\mathbf{M}(\mu,\nu,z)=\exp\left(-\frac{z}{2}\right)z^{\frac{1}{2}+\nu}\,{}_1F_1(\frac{1}{2}+\nu-\mu,1+2\nu,z)$ is the Whittaker function, $a\neq 1/3$, $a\neq 1$.
- The relative entropy tends to its maximum value at infinite z_b: $s\to$
$$\frac{\left(\frac{L}{z_a}\right)^{3a} z_a\exp\left(-\frac{3{z_a}^2}{2R^2}\right)\left(2\left(\frac{3z_a^2}{R^2}\right)^{\frac{3a-1}{4}}\mathbf{M}\left(\frac{-3a+1}{4},\frac{3(-a+1)}{4},\frac{3{z_a}^2}{R^2}\right)+3(1-a)\exp\left(-\frac{3{z_a}^2}{2R^2}\right)\right)}{6G_5\cdot(3\,a-1)(1-a)},$$
where $a>\frac{1}{3}$, $a\neq 1$, z_a is defined by
$$z_a|_{z_b\to\infty}=R\sqrt{\frac{a}{2}}\sqrt{\mathrm{W}\left(\frac{2L^2}{aR^2}\left(\frac{L^2}{8\pi G_5E}\right)^{\frac{2}{3a}}\right)}.$$

Thus, the entropy can be roughly estimate at a=1/2 such as
$S \sim E^{0.3}(1 + C_1 \ln(E + 100)) - C_2$

- $C_1 = -0.738,\ C_2 = 0.393$ at $10 < E < 100$ GeV
- $C_1 = -0.073,\ C_2 = 0.827$ at $100 < E < 1000$ GeV

The dependence of the maximum relative entropy on energy (solid line) and its approximation



(crosses) at the parameter value $a = 1/2$: (a) approximation $(E^{0.3}(57 - 29.75(\log(E+100))) - 7)/2G_5$ in the energy interval $0 < E < 10$ GeV, (b) approximation $(E^{0.3}(61 - 45.05(\log(E+100))) - 24)/2G_5$ in the energy interval $10 < E < 100$ GeV, and (c) approximation $(E^{0.3}(81 - 5.95\log(E+100)) - 67)/2G_5$ in the energy interval $10^2 < E < 10^3$ GeV.

Mixed b-factor $b(z) = \left(\frac{L}{z}\right)^a \exp\left(-z^2/R^2\right)$, potential

For this b-factor we can express $V(\Phi(z))$ and $\Phi(z)$ as

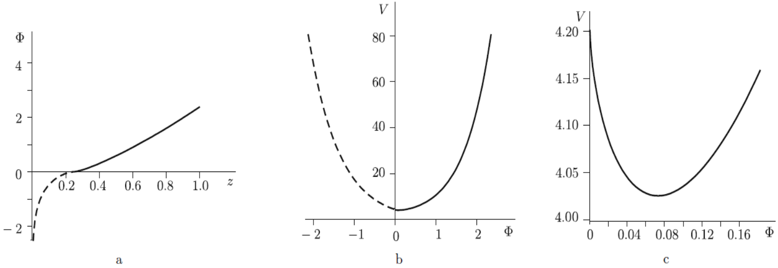
$$V(z) = -\frac{12}{L^2} + \frac{3\left(\frac{L}{z}\right)^{-2a}\left(aR^4(3a+1) + 2z^2R^2(6a-1) + 12z^4\right)\exp\left(\frac{2z^2}{R^2}\right)}{z^2R^4},$$

$$\Phi_{\pm} = \pm\left(\frac{3}{4}\frac{\xi}{R^2} + \frac{3}{8}(2a+1)\ln\left(\xi + \frac{(2a+1)R^2 + 4z^2}{2}\right)\right.$$
$$\left. - \frac{3}{4}\sqrt{a(a-1)}\ln\left(\frac{2R^2\left\{a(a-1)R^2 + (2a+1)z^2 + \xi\sqrt{a(a-1)}\right\}}{z^2}\right)\right) + \Phi_{0\pm}.$$

$\zeta = 4z^4 + 2R^2(2a+1)z^2 + aR^4(a-1)$.
We can find $V(\Phi)$ at $z \to \infty$, $\Phi \sim \frac{3}{2}\frac{z^2}{R^2}$ and $V \sim \Phi^{a+1}\,e^{\frac{4}{3}\Phi}$.

- $\Phi(z)$ is real for $z > z_0$, $(\zeta(z_0) = 0)$
- and becomes imaginary for $z < z_0$, $(\Phi_{\pm} - \Phi_{0\pm}) = i(\Phi_{p\pm} - \Phi_{p0\pm})$

The convenient choice of constants is $\Phi_{s\pm}(z_0) = \Phi_{p\pm}(z_0) = 0$. The imaginary scalar field corresponds to the phantom sign of the kinetic term and one can write $\Phi = \Phi_s \Theta(z - z_0) + i\Phi_p \Theta(z_0 - z)$ and interpret this model as a model with an alternating sign of the kinetic term.



The plots corresponding to a = 1/2, L = 4.4 fm, R=1 fm. A. The phantom Φp (dashed line) and dilaton Φs (solid line) fields as functions of z. B. The dependence of the potential V on the dilaton and the phantom fields. C. The same dependence of the potential V on the dilaton field as in B for small Φ.

Conclusions

- The black holes formation in the domain wall-wall collisions is investigated in the modified AdS_5 spaces with b-factors
- We analyzed the dependence of entropy on the energy of colliding ions in the spaces with b-factors based on the analysis of the conditions for forming the trapped surfaces.
- With the AdS/CFT duality taken into account, the obtained results allow modeling the dependence of multiplicity of the produced particles on the energy of the colliding heavy-ions.
- The derived results can be used to compare with the experimental curves for the multiplicity of particle formation in heavy-ion collisions.
- In the cases of good agree with experiment ($a < 1$), we found that in the spaces with power-law b-factor the scalar field is phantom one and in the space with the modernized mixed b-factor $b = (L/z)^a e^{-z^2/R^2}$ the scalar field is phantom at the interval $z < z_0$ and dilaton at the interval $z > z_0$.

Thank you for attention!