COSMOLOGICAL CONSTANT PROBLEM IN SPINCUBE MODELS OF QUANTUM GRAVITY

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CC IN CLASSICAL THEORY

Classical theory of matter fields in flat spacetime is invariant wrt.

$$\mathcal{L}_M(\eta, \phi, \partial\phi) \to \mathcal{L}_M(\eta, \phi, \partial\phi) + C, \qquad \text{for arbitrary } C.$$

Upon coupling to gravity, the equivalence principle transforms this ambiguity to the (classical, bare) cosmological constant,

$$S[g, \phi] = -\frac{1}{16\pi l_p^2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_M(g, \phi, \nabla\phi) + \frac{1}{8\pi l_p^2} \int d^4x \sqrt{-g} \Lambda_b$$

$(C \equiv \Lambda_b / 8\pi l_p^2)$, **which enters the Einstein field equations:**

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda_b = 8\pi l_p^2 \, T_{\mu\nu}(\phi).$$

The value of Λ_b is *completely arbitrary*, due to the above ambiguity of the equivalence principle.

CC IN CLASSICAL THEORY

In the Standard Model coupled to gravity there are three dimensionful parameters:

- the Planck length l_p, fixing the gravitational scale,
- the cosmological constant Λ_{eff}, determining the cosmological scale, and
- the Higgs mass m_H, determining the electroweak scale.

Taking the ratios wrt. Planck length, we obtain:

$$c_\Lambda \equiv \Lambda_{\text{eff}} l_p^2 \approx 10^{-122} \quad \leftarrow \text{ CC problem!}$$
$$c_H \equiv m_H^2 l_p^2 \approx 10^{-34} \quad \leftarrow \text{ Hierarchy problem!}$$

All other coupling constants in the SM are $\lesssim 1$, which makes c_Λ and c_H *unusually small*. A natural question to ask is:

WHY DOES THIS HAPPEN?

CC IN QUANTUM FIELD THEORY

Quantization of matter fields (keeping gravity classical) makes things even worse:

- calculate the expectation value of the stress-energy tensor at one-loop order,

$$\langle \hat{T}_{\mu\nu}(\phi) \rangle = T_{\mu\nu}^{\text{classical}}(\phi) + T_{\mu\nu}^{\text{1-loop}}(\phi),$$

- evaluate stress-energy for the ground state of matter fields, $\phi = 0$,

$$\begin{aligned} \langle \hat{T}_{\mu\nu}(\phi) \rangle \Big|_{\phi=0} &= \underbrace{T_{\mu\nu}^{\text{classical}}(0)}_{0} + T_{\mu\nu}^{\text{1-loop}}(0) = \Big[\text{textbook by Birrell\&Davies} \Big] = \\ &= \frac{a_1}{l_p^4} g_{\mu\nu} + \frac{a_2}{l_p^2} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + a_3 \left(\dots R^2 \dots \right) + \dots, \end{aligned}$$

  where $a_1, a_2, a_3, \dots$ are dimensionless constants of $\mathcal{O}(1)$,

- substitute into Einstein equations and read off the renormalized value of the CC:

$$\Lambda_{\text{eff}} = \Lambda_b + \Lambda_m, \qquad \text{where} \qquad \Lambda_m \equiv -\frac{8\pi a_1}{l_p^2}.$$

CC IN QUANTUM FIELD THEORY

Why is this even worse:

$$\begin{array}{ccccc} \Lambda_{\rm eff} l_p^2 & = & \Lambda_b l_p^2 & - & 8\pi a_1 \\ \uparrow & & \uparrow & & \uparrow \\ \mathcal{O}(10^{-122}) & & \text{arbitrary} & & \mathcal{O}(1) \end{array}$$

In order to satisfy this equation, one needs to choose Λ_b to arrange for the cancellation of type:

CC IN QUANTUM FIELD THEORY

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$$
\begin{aligned}
(1.\cdots) - (1.\cdots) \;=\; & 0.0000000000000000000000000 \\
& 0000000000000000000000000 \\
& 0000000000000000000000000 \\
& 0000000000000000000000000 \\
& 0000000000000000000001\cdots
\end{aligned}
$$

EXTREME FINE TUNING !!!

CC IN QUANTUM GRAVITY

Fundamental assumption for quantum gravity construction:

Nature has a physical cutoff at the Planck scale!

The spincube model of quantum gravity:

(1) Rewrite GR action as a topological $BFCG$ theory plus constraint:

$$S = \int \underbrace{B_{ab} \wedge R^{ab} + e^a \wedge G_a}_{\text{topological sector}} - \underbrace{\phi_{ab} \left(B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right)}_{\text{constraint}} ,$$

(2) Quantize the theory by

- triangulating the spacetime manifold,
- defining the path integral on the triangulation for the topological sector,
- enforcing the constraint,
- redefining the measure so that the theory is finite and has a well-defined classical limit.

(3) Introduce matter fields on the triangulation in a straightforward way.

CC IN QUANTUM GRAVITY

After the dust settles, one ends up with:

$$Z = \int \mathcal{D}g \int \mathcal{D}\phi \; e^{iS[g,\phi]} \stackrel{\text{def}}{=} \prod_{\epsilon \in T(\mathcal{M})} \int_0^\infty dL_\epsilon \; \mu(L) \prod_{r \in T(\mathcal{M})} \int_{-\infty}^{\infty} d\phi_r \; e^{iS[L,\phi]}.$$

Calculation of the semiclassical limit, $l_p \to 0$, involves the following integrals:

$$\int_{-\infty}^{\infty} dx \, e^{-x^2/l_p^2} = l_p \sqrt{\pi} \quad \text{and} \quad \int_{-L}^{\infty} dx \, e^{-x^2/l_p^2} = l_p \sqrt{\pi} \left[1 - \frac{l_p}{2L\sqrt{\pi}} e^{-L^2/l_p^2} + \ldots \right].$$

Having a well-defined classical limit requires the suppression of the nonanalytic terms, which can be achieved only with an exponential measure:

$$\mu(L) = \exp\left(-\frac{1}{8\pi l_p^2} \Lambda_\mu V_4[L] \right), \qquad \text{where} \qquad 0 < \Lambda_\mu \ll \frac{1}{l_p^2}.$$

CRUCIAL PROPERTY OF QG KINEMATICS !!!

CC IN QUANTUM GRAVITY

How to calculate CC in QG? The effective action equation in QFT:

$$e^{i\mathit{\Gamma}[\phi]} = \int \mathcal{D}\chi \exp\left[iS[\phi+\chi] - i\int d^4x \frac{\delta\mathit{\Gamma}[\phi]}{\delta\phi}\chi\right].$$

This can be generalized to QG in a straightforward manner:

$$e^{i\mathit{\Gamma}[L,\phi]} = \int \mathcal{D}l\ \mu(L+l) \int \mathcal{D}\chi \exp\left[iS[L+l,\phi+\chi] - i\int d^4x \left(\frac{\delta\mathit{\Gamma}}{\delta L}l + \frac{\delta\mathit{\Gamma}}{\delta\phi}\chi\right)\right].$$

The classical action has the form

$$S[L,\phi] = S_R[L] + S_M[L,\phi] + \frac{1}{8\pi l_p^2}\Lambda_b V_4[L],$$

and matter fields are in the ground state,

$$\phi = 0, \qquad \frac{\delta\mathit{\Gamma}}{\delta\phi} = 0.$$

Effective action equation reduces to:

$$e^{i\mathit{\Gamma}[L,0]} = \int \mathcal{D}l\ e^{iS_R[L+l] + \frac{i}{8\pi l_p^2}(\Lambda_b + i\Lambda_\mu)V_4[L+l] - i\int \frac{\delta\mathit{\Gamma}}{\delta L}l} \int \mathcal{D}\chi\, e^{iS[L+l,\chi]}.$$

CC IN QUANTUM GRAVITY

The matter path integral can be evaluated perturbatively,

$$\int \mathcal{D}\chi \, e^{iS[L+l,\chi]} = \exp\left[-i\frac{a_1}{l_p^4}V_4[L+l] - ia_2 S_R[L+l] - ia_3 l_p^2 S(...R^2...) + \ldots\right],$$

so we obtain the effective action equation:

$$\begin{aligned} e^{i\Gamma[L,0]} \; = \; \int \mathcal{D}l \exp\left[\frac{i}{8\pi l_p^2}\left(\Lambda_b + i\Lambda_\mu - \frac{8\pi a_1}{l_p^2}\right)V_4[L+l] + i(1-a_2)S_R[L+l] \right. \\ \left. -ia_3 l_p^2 S(...R^2...) - i\int d^4x \, \frac{\delta\Gamma}{\delta L} l\right]. \end{aligned}$$

Performing the semiclassical limit and Wick rotation, we obtain

$$\Gamma[L,0] = \frac{1}{8\pi l_p^2}\Lambda_{\text{eff}} V_4[L+l] + (1-a_2)S_R[L+l] + \mathcal{O}(l_p^2),$$

where the effective CC is given as:

$$\Lambda_{\text{eff}} \equiv \Lambda_b - \frac{8\pi a_1}{l_p^2} + \Lambda_\mu.$$

CC IN QUANTUM GRAVITY

Fit to experimental data:

$$\begin{array}{ccccccc} \Lambda_{\text{eff}} l_p^2 & = & \Lambda_b l_p^2 & - & 8\pi a_1 & + & \Lambda_\mu l_p^2. \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \mathcal{O}(10^{-122}) & & \text{arbitrary} & & \mathcal{O}(1) & & 0 < \ldots \ll 1 \end{array}$$

We are free to choose exact cancellation of classical and matter contributions (NO FINE TUNING !!!),

$$\Lambda_b \stackrel{\text{def}}{=} \frac{8\pi a_1}{l_p^2}, \qquad \Rightarrow \qquad \Lambda_{\text{eff}} = \Lambda_\mu,$$

so that

$$0 < 10^{-122} \ll 1.$$

NONZERO VALUE OF THE CC IS A PURE QUANTUM GRAVITY EFFECT !!!

CONCLUSIONS

Our assumptions:

- **nature has a physical cutoff at the Planck scale**

  $\Rightarrow$ physical triangulation of spacetime,

- **spincube quantization procedure for gravity**

  $\Rightarrow$ edge-lengths are fundamental degrees of freedom,

- **quantum fluctuations of matter fields do not gravitate**

  $\Rightarrow$ classical CC exactly cancels vacuum fluctuations.

Consequence:

- **cosmological constant is a quantum gravity effect**

  $\Rightarrow$ good agreement with experiment: $0 < 10^{-122} \ll 1.$

THANK YOU!